SELFDUAL SPIN 2 THEORY IN A 2+1 DIMENSIONAL CONSTANT CURVATURE SPACE-TIME

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Abstract

The Lagrangian constraint analysis of the selfdual massive spin 2 theory in a 2+1 dimensional flat space-time and its extension to a curved one, are performed. Demanding consistence of degrees of freedom in the model with gravitational interaction, gives rise to physical restrictions on non minimal coupling terms and background. Finally, a constant curvature scenario is explored, showing the existence of forbidden mass values. Causality in these spaces is discussed. Aspects related with the construction of the reduced action and the one-particle exchange amplitude, are noted.

In the context of ordinary field theory, there has been great interest about the lagrangian study of higher spin fields with external interaction[1]-[11]. These theories are only known in certain backgrounds (i.e., constant curvature, non Einstein's spaces), because, in general, a consistent higher spin field theory with interaction does not exist as a result of the no conservation of the degrees of freedom and causality violation.

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It is well known[12, 13] that the introduction of auxiliary fields, that vanish on shell is needed in order to obtain a lagrangian formulation, in a flat space-time, for a massive field with spin s, without interaction, described by a symmetric, transverse and traceless, rank s, tensor (i.e., $\partial^\mu h_{\mu\mu_1\ldots\mu_{s-1}} = 0, \;\; h^\mu{}_{\mu\mu_1\ldots\mu_{s-2}} = 0$). However, when an arbitrary interaction is turned on, auxiliary fields become dynamic and hence they could modify the number of local degrees of freedom.

Causality violation has also been noted[11, 14, 15]. For this, let us use the following notation[11]: The equations of motion for an integer spin field, $h_{\alpha_1\alpha_2\ldots}$, which come from some lagrangian formulation, can be written as $(\mathcal{M}_{\beta_1\beta_2\ldots\alpha_1\alpha_2\ldots})^{\mu\nu}\nabla_\mu\nabla_\nu h^{\alpha_1\alpha_2\cdots} + \ldots = 0$, with the help of the lagrangian constraints[16]. Let the vector n_μ, used to define the characteristic matrix $\mathcal{M}_{AB}(n) \equiv \mathcal{M}_{AB}{}^{\mu\nu} n_\mu n_\mu$, where A, B are composed indexes. The solutions of the characteristic equation $\det \mathcal{M}_{AB}(n) = 0$, define characteristic surfaces that might describe some propagation process. If the solution of the characteristic equation gives rise to a real n_0, the system is called hyperbolic. An hyperbolic system is called causal if there is no time like vectors among the solutions of the characteristic equation (on the contrary, if there exists time like vectors, the corresponding characteristic surfaces are space like and violate causality). When an arbitrary external interaction is considered, the characteristic matrix, $\mathcal{M}_{AB}(n)$ does not necessarily define an hyperbolic-causal equation of motion system.

In this work we are interested in the study of some dinstinctive features related to the aforementioned problems in the lagrangian formulation of the selfdual massive spin 2 field in $2+1$ dimensions[17, 18], coupled with gravity[19]. This letter is organized as follows. We will start with a brief review of the lagrangian constraints analysis of selfdual massive spin 2 theory in a flat space-time without external interaction. Next, we introduce the coupling between selfdual massive spin 2 field with an arbitrary gravitational background, through a suitable set of non minimal terms in the lagrangian formulation, and we will discuss the physical restrictions that arise in order to preserve a consistent interaction. As it is expected, one can find a constant curvature space solution, in which the degrees of freedom must be consistently preserved and causality must take place. There, we obtain forbidden mass values for the selfdual massive spin 2 field. The construction of the reduced action in constant curvature space-time is noted. At the end we discuss the one-particle exchange amplitude. Finally, some remarks will be stated.

The action of selfdual massive spin 2 field[17], in flat space-time is

$$S_{sd} = \int \frac{d^3x}{2}(m\,\epsilon^{\mu\nu\lambda} h_\mu{}^\alpha \partial_\nu h_{\lambda\alpha} - m^2\,(h_{\mu\nu}h^{\nu\mu} - h^2)) \ , \tag{1}$$

where $h = h^\mu{}_\mu$, $\epsilon^{012} \equiv \epsilon^{12} = +1$, and Minkowski's metric is $diag(-++)$. Equation of motion coming from S_{sd}, provide nine primary constraints

$$\phi^{(1)\mu\rho} = m\,\epsilon^{\mu\nu\lambda}\partial_\nu h_\lambda{}^\rho + m^2(\eta^{\mu\rho}h - h^{\rho\mu}) \approx 0 \ . \tag{2}$$

Preservation of (2), take us to the secondary constraints

$$\phi^{(2)\rho} \equiv \dot{\phi}^{(1)0\rho} \approx \partial_\mu \phi^{(1)\mu\rho} \equiv m^2\partial^\rho h - m^2\partial_\mu h^{\rho\mu} \approx 0 \ . \tag{3}$$

We observe that (3) can be replaced with the combination $\phi^{(2)\rho} \approx \partial_\mu\phi^{(1)\mu\rho} - m\epsilon^{\rho\mu\alpha}\phi^{(1)}{}_{\mu\alpha} \equiv -m^3\epsilon^{\rho\mu\alpha}h_{\mu\alpha} \approx 0$, which enforces $h_{\mu\nu}$ to be symmetric. Relations, $\dot{\phi}^{(1)i\rho} = 0$ allow us to find the following accelerations

$$\ddot{h}_{k\rho} = \partial_k \dot{h}_{0\rho} + m\epsilon_{ik}(\delta^i{}_\rho \dot{h} - \dot{h}_\rho{}^i) \ , \tag{4}$$

and the $\ddot{h}_{0\rho}$ remain unknown.

Procedure continues with the preservation of $\phi^{(2)\rho} \approx 0$, that gives rise to three additional constraints

$$\phi^{(3)\rho} \equiv \dot{\phi}^{(2)\rho} \approx -m^3\epsilon^{\rho\mu\alpha}\dot{h}_{\mu\alpha} \approx 0 \ , \tag{5}$$

saying that the symmetry property is consistent with time evolution. If we look at (5), the $\rho = 0$ component can be rewritten, on shell, as $\phi^{(3)0} \approx \partial_\rho\phi^{(2)\rho} \equiv -m^3\epsilon^{\rho\mu\alpha}\partial_\rho h_{\mu\alpha} \approx 2m^4 h \approx 0$, which shows the traceless property of the tensor field. Then, preserving $\phi^{(3)\rho} \approx 0$ we obtain the last constraint

$$\phi^{(4)} \equiv 2m^4\dot{h} \approx 0 \ , \tag{6}$$

and two relations for the remaining accelerations, $-m^3\epsilon^{k\mu\alpha}\ddot{h}_{\mu\alpha}=0$. This allow us to obtain

$$\ddot{h}_{0k}=\ddot{h}_{k0}=\partial_k\dot{h}_{00}+m\epsilon_{ik}(\delta^i{}_0\dot{h}-\dot{h}_0{}^i)\ . \tag{7}$$

The analysis of the lagrangian constraints ends whith the preservation of (6). This provides one more relation for the accelerations, $m^4\ddot{h}=0$ from which we obtain

$$\ddot{h}_{00}=-\ddot{h}_{ii}=-\partial_k\dot{h}_{0k}-m\epsilon_{ik}(\delta^i{}_k\dot{h}-\dot{h}_k{}^i)\ . \tag{8}$$

So, it can be shown that the 16 lagrangian constraints indicate the existence of one propagated excitation, and it is described by a symmetric, transverse and traceless tensor field. In other words

$$h^{(s)Tt}{}_{\mu\nu}=h^{(s)Tt}{}_{\nu\mu}\ ,\quad \partial^\mu h^{(s)Tt}{}_{\mu\nu}=0\ ,\quad h^{(s)Tt}{}_\mu{}^\mu=0\ , \tag{9}$$

respectively, satisfying the field equation

$$\epsilon^{\mu\nu\lambda}\partial_\nu h^{(s)Tt}{}_\lambda{}^\rho-mh^{(s)Tt\rho\mu}=0\ . \tag{10}$$

A Klein-Gordon type equation, $(\Box-m^2)h^{(s)Tt}{}_{\mu\nu}=0$ is obtained from (10) using (9).

It can be observed that restrictions (9) leave just two free components of the nine in $h_{\mu\nu}$, but relying to dynamic restriction (10), it can be seen that only one degree of freedom is locally propagated. From the action point of view, one can also expose this unique excitation through the construction of the reduced action (S^*_{sd}), which starts performing a $2+1$ splitting for $h_{\mu\nu}$, this means, $n=h_{00},\ N_i=h_{i0},\ M_i=h_{0i},\ h^{(s)}{}_{ij}=\frac{1}{2}(h_{ij}+h_{ji}),\ V=\frac{1}{2}\epsilon_{ij}h_{ij}$, in action (1)[20]. Then, a transverse-longitudinal decomposition is realized introducing new variables defined by: $N_i\equiv\epsilon_{ik}\partial_kN^T+\partial_iN^L\ ,\ M_i\equiv\epsilon_{ik}\partial_kM^T+\partial_iM^L\ ,\ h^{(s)}{}_{ij}\equiv(\delta_{ij}\Delta-\partial_i\partial_j)h^{TT}+\partial_i\partial_jh^{LL}+(\epsilon_{ik}\partial_k\partial_j+\epsilon_{jk}\partial_k\partial_i)h^{TL}$. This decomposition establishes an easy way to obtain the reduced action using the corresponding field equations, $S^*_{sd}=\int d^3x\{P\dot{Q}-\frac{1}{2}P^2+\frac{1}{2}Q(\Delta-m^2)Q\}$, where $Q\equiv\sqrt{2}\,\Delta h^{TT}$ and $P\equiv\sqrt{2}\,m\Delta h^{TL}$, which describes a single massive mode.

Now we outline the model of selfdual massive spin 2 field non minimally coupled with gravity in a torsionless space-time[19] as follows

$$S_{sdg} = \int \frac{d^3x}{2} \sqrt{-g} (m\, \varepsilon^{\mu\nu\lambda} h_\mu{}^\alpha \nabla_\nu h_{\lambda\alpha} + \Omega^{\alpha\beta\sigma\lambda} h_{\alpha\beta} h_{\sigma\lambda}) \ , \tag{11}$$

where ∇_ν is the covariant derivative and $\varepsilon^{\mu\nu\lambda} \equiv \frac{\epsilon^{\mu\nu\lambda}}{\sqrt{-g}}$. Due to the fact that in $2+1$ dimensions the Riemann curvature tensor can be written in terms of the Ricci tensor (i.e., $R_{\lambda\mu\nu\rho} = g_{\lambda\nu} R_{\mu\rho} - g_{\lambda\rho} R_{\mu\nu} - g_{\mu\nu} R_{\lambda\rho} + g_{\mu\rho} R_{\lambda\nu} - \frac{R}{2} (g_{\lambda\nu} g_{\mu\rho} - g_{\lambda\rho} g_{\mu\nu})$), so the non minimal coupling in (11) is characterized by a tensor $\Omega^{\alpha\beta\sigma\lambda}$, whose general form is

$$\begin{aligned}\Omega^{\alpha\beta\sigma\lambda} \equiv{}& m^2 (g^{\sigma\lambda} g^{\alpha\beta} - g^{\sigma\beta} g^{\alpha\lambda}) + a_1 (R^{\sigma\lambda} g^{\alpha\beta} + R^{\alpha\beta} g^{\sigma\lambda}) + a_2 (R^{\sigma\beta} g^{\alpha\lambda} + R^{\alpha\lambda} g^{\sigma\beta}) \\ &+ a_3 R^{\alpha\sigma} g^{\beta\lambda} + a_4 R^{\beta\lambda} g^{\alpha\sigma} + a_5 R g^{\alpha\beta} g^{\sigma\lambda} + a_6 R g^{\sigma\beta} g^{\alpha\lambda} + a_7 R g^{\lambda\beta} g^{\sigma\alpha} \ ,\end{aligned} \tag{12}$$

with the property $\Omega^{\alpha\beta\sigma\lambda} = \Omega^{\sigma\lambda\alpha\beta}$ and real parameters a_n, $n = 1, ..., 7$.

Taking arbitrary variations on $h_{\mu\nu}$ in S_{sdg} gives rise to the following field equations

$$\Phi^{(1)\mu\alpha} \equiv m\, \varepsilon^{\mu\nu\lambda} \nabla_\nu h_\lambda{}^\alpha + \Omega^{\mu\alpha\sigma\lambda} h_{\sigma\lambda} \approx 0 \ , \tag{13}$$

wich constitute nine primary constraints.

Three more constraints arise when $\Phi^{(1)o\rho} \approx 0$ is preserved

$$\Phi^{(2)\alpha} \approx \nabla_\mu \Phi^{(1)\mu\alpha} \equiv \Omega^{\mu\alpha\sigma\lambda} \nabla_\mu h_{\sigma\lambda} + \mathcal{B}^{\alpha\sigma\lambda} h_{\sigma\lambda} \approx 0 \ , \tag{14}$$

where

$$\mathcal{B}^{\alpha\sigma\lambda} \equiv \frac{m}{2}\, \varepsilon^{\mu\nu\rho} (R^{\alpha\lambda}{}_{\nu\mu}\, \delta^\sigma{}_\rho - R^\sigma{}_{\rho\nu\mu}\, g^{\alpha\lambda}) + \nabla_\mu \Omega^{\mu\alpha\sigma\lambda} \ . \tag{15}$$

On the other hand, preservation of $\Phi^{(1)k\alpha} \approx 0$ leads to six relations for the accelerations ($m \neq 0$, as in the flat case)

$$\begin{aligned}\nabla_0{}^2 h_j{}^\alpha = {}& -\frac{\varepsilon_{0kj}}{m}\, \Omega^{k\alpha\sigma\lambda} \nabla_0 h_{\sigma\lambda} - \big(\frac{\varepsilon_{0kj}}{m}\, \nabla_0 \Omega^{k\alpha\sigma\lambda} + R^\sigma{}_{0j0}\, g^{\alpha\lambda}\big) h_{\sigma\lambda} + \\ &+ \nabla_j \nabla_0 h_0{}^\alpha + R^{\alpha\lambda}{}_{j0}\, h_{0\lambda} \ ,\end{aligned} \tag{16}$$

remaining the unknown accelerations, ${\nabla_0}^2 h_{0\lambda}$.

At this point, the lagrangian analysis with free coupling parameters in arbitrary background is equivalent to the flat space-time case. The next step is the preservation of the constraint $\Phi^{(2)\alpha} \approx 0$, which leads to

$$\begin{aligned}\Phi^{(3)\alpha} \equiv \nabla_0 \Phi^{(2)\alpha} \approx \Omega^{0\alpha 0\lambda} {\nabla_0}^2 h_{0\lambda} + \big(\Omega^{0\alpha j\lambda} + \Omega^{j\alpha 0\lambda}\big)\nabla_j \nabla_0 h_{0\lambda} + \\ + \big(- \frac{\varepsilon_{0kl}}{m}\, \Omega^{0\alpha l\rho} \Omega^{\mu\lambda k}{}_{\rho} + \nabla_0 \Omega^{0\alpha\mu\lambda} + \mathcal{B}^{\alpha\mu\lambda}\big)\nabla_0 h_{\mu\lambda} + \\ + \big[- \frac{\varepsilon_{0kl}}{m}\, \Omega^{0\alpha l\rho} \nabla_0 \Omega^{\mu\lambda k}{}_{\rho} - \Omega^{0\alpha l\lambda} R^{\mu}{}_{0l0} + \nabla_0 \mathcal{B}^{\alpha\mu\lambda} + \\ - \Omega^{\nu\alpha\mu\rho} R^{\lambda}{}_{\rho\nu 0} - \Omega^{\sigma\alpha\nu\lambda} R^{\mu}{}_{\nu\sigma 0}\big] h_{\mu\lambda} - \Omega^{0\alpha l\rho} R^{\lambda}{}_{\rho l 0} h_{0\lambda} + \\ + \nabla_0 \Omega^{i\alpha\mu\lambda} \nabla_i h_{\mu\lambda} + \Omega^{i\alpha j\lambda} \nabla_i \nabla_0 h_{j\lambda} \approx 0 \ ,\end{aligned} \tag{17}$$

and we expect that this expression represents three additional constraints, as in the flat case. But, from (17) it would be impossible to obtain any relation for the accelerations ${\nabla_0}^2 h_{0\lambda}$, because (17) constitutes a complete system for the aforementioned accelerations. We demand that all matrixes 3×3, 2×2 and 1×1 built with $\Omega^{0\alpha 0\lambda}$, have null determinant (i.e., $\Omega^{0\alpha 0\lambda}$ totally degenerated), in other words

$$\Omega^{0\alpha 0\lambda} = 0 \ . \tag{18}$$

This condition gives rise to restrictions on coupling parameters. Using (12)

$$a_1 = -a_2 \equiv a \ , \ a_6 = -a_5 \equiv b \ , \ a_3 = a_4 = a_7 = 0 \ , \tag{19}$$

and just two free parameters remain. Then,

$$\Omega^{\alpha\beta\sigma\lambda} = a\, R^{\alpha\sigma\beta\lambda} + (m^2 + (\frac{a}{2} - b) R)(g^{\alpha\beta} g^{\sigma\lambda} - g^{\sigma\beta} g^{\alpha\lambda}) \ , \tag{20}$$

and now

$$\Omega^{\alpha\beta\sigma\lambda} = \Omega^{\sigma\lambda\alpha\beta} = -\Omega^{\alpha\lambda\sigma\beta} \ . \tag{21}$$

The object $\mathcal{B}^{\alpha\sigma\lambda}$, can be rewritten in terms of the Einstein's tensor, $G_{\mu\nu} = R_{\mu\nu} - \frac{g_{\mu\nu}}{2}\, R$ as follows

$$\mathcal{B}^{\alpha\sigma\lambda} \equiv m\, \varepsilon^{\alpha\lambda\beta}\, G^{\sigma}{}_{\beta} + \nabla_{\mu}\Omega^{\mu\alpha\sigma\lambda} = -\mathcal{B}^{\lambda\sigma\alpha} \ , \tag{22}$$

with an antisymmetric property in virtue of (21).

With the help of (20) we can write the three constraints, $\Phi^{(3)\rho} \equiv \nabla_0 \Phi^{(2)\rho} \approx 0$ as follows

$$\begin{aligned} \Phi^{(3)\alpha} = \mathcal{N}^{\alpha\lambda}\nabla_0 h_{0\lambda} + \nabla_0\Omega^{i\alpha 0\lambda}\nabla_i h_{0\lambda} + \mathcal{A}^{\alpha\lambda} h_{0\lambda} + \Omega^{i\alpha j\lambda}\nabla_i\nabla_0 h_{j\lambda} \\ + \mathcal{C}^{\alpha j\lambda}\nabla_0 h_{j\lambda} + \nabla_0\Omega^{i\alpha j\lambda}\nabla_i h_{j\lambda} + \mathcal{D}^{\alpha j\lambda} h_{j\lambda} \approx 0 \ , \end{aligned} \tag{23}$$

where

$$\mathcal{N}^{\alpha\lambda} \equiv \frac{1}{m}\, \varepsilon_{0kl}\, \Omega^{0\alpha k\rho}\Omega^{0\lambda l}{}_{\rho} + \mathcal{B}^{\alpha 0\lambda} = -\mathcal{N}^{\lambda\alpha} \ , \tag{24}$$

$$\begin{aligned} \mathcal{A}^{\alpha\lambda} \equiv -\frac{1}{m}\, \Omega^{0\alpha l\rho}\big(\varepsilon_{0kl}\, \nabla_0\Omega^{0\lambda k}{}_{\rho} + mR^{\lambda}{}_{\rho l0} + mR^{0}{}_{0l0}\delta^{\lambda}{}_{\rho}\big) \\ + \nabla_0\mathcal{B}^{\alpha 0\lambda} - \Omega^{\mu\alpha 0\rho} R^{\lambda}{}_{\rho\mu 0} - \Omega^{\mu\alpha\nu\lambda} R^{0}{}_{\nu\mu 0} \ , \end{aligned} \tag{25}$$

$$\mathcal{C}^{\alpha j\lambda} \equiv -\frac{1}{m}\, \varepsilon_{0kl}\, \Omega^{0\alpha l\rho}\Omega^{j\lambda k}{}_{\rho} + \mathcal{B}^{\alpha j\lambda} + \nabla_0\Omega^{0\alpha j\lambda} \ , \tag{26}$$

$$\begin{aligned} \mathcal{D}^{\alpha j\lambda} \equiv -\frac{1}{m}\, \varepsilon_{0kl}\, \Omega^{0\alpha l\rho}\nabla_0\Omega^{j\lambda k}{}_{\rho} + \nabla_0\mathcal{B}^{\alpha j\lambda} - \Omega^{\mu\alpha j\rho} R^{\lambda}{}_{\rho\mu 0} \\ -\Omega^{\mu\alpha\nu\lambda} R^{j}{}_{\nu\mu 0} - \Omega^{0\alpha l\lambda} R^{j}{}_{0l0} \ . \end{aligned} \tag{27}$$

Going on with the lagrangian procedure, preservation of $\Phi^{(3)\rho} \approx 0$ must represents, as in the flat case, two expressions for accelerations ${\nabla_0}^2 h_{0\sigma}$ and one for the last constraint (whose preservation allow us to get the remaining accelerations, and the procedure ends). Let us consider 3×3 and 2×2 arrays built with objects $\mathcal{N}^{\alpha\lambda}$, the last request means that

$$\det(\mathcal{N}^{\alpha\lambda}) = 0 \ , \tag{28}$$

$$\det(\mathcal{N}^{ij}) \neq 0 \ . \tag{29}$$

Relation (28) due to the antisymmetry property of the odd rank matrix ($\mathcal{N}^{\alpha\lambda}$)is identically satisfied. (29) gives a physical restriction on the gravitational field, and it conduces to

$$\varepsilon_{0ij}\,\mathcal{N}^{ij} \neq 0 \ . \tag{30}$$

It can be shown that this restriction, that should be satisfied in order to keep consistence in the number of degrees of freedom, will include non Einsteinian solutions. Although these type of solutions exist, (30) will impose conditions on them. For illustration, let consider $R_{\lambda\mu\nu\rho} = \frac{f(x)}{6}\,(g_{\lambda\nu}g_{\mu\rho} - g_{\lambda\rho}g_{\mu\nu})$. Then, restriction (30) enforce a constraint for $f(x)$ (i.e., $6M^4 - m^2 f(x) + m\sigma\varepsilon^k{}_0\partial_k f(x) \neq 0$, with $\sigma \equiv \frac{2}{3}\,a - b$). Our interest is focussed in the particular solution $\partial_\mu f(x) = 0$ (hence (30) relates the mass with cosmological constant), which is of dS/AdS type.

Considering a constant curvature space-time, with cosmological constant λ, been related to a dS space ($\lambda > 0$) or AdS space ($\lambda < 0$) via Einstein's equation, $R_{\mu\nu} - \frac{g_{\mu\nu}}{2}\,R - \lambda\, g_{\mu\nu} = 0$, where Riemann and Ricci tensors are

$$R_{\lambda\mu\nu\rho} = \frac{R}{6}\,(g_{\lambda\nu}g_{\mu\rho} - g_{\lambda\rho}g_{\mu\nu}) \ , \quad R_{\mu\nu} = \frac{R}{3}\,g_{\mu\nu} \ , \tag{31}$$

respectively, and

$$R = -6\lambda \ . \tag{32}$$

(20) is

$$\Omega^{\alpha\beta\sigma\lambda} = M^2\,(g^{\alpha\beta}g^{\sigma\lambda} - g^{\sigma\beta}g^{\alpha\lambda}) \ , \tag{33}$$

where

$$M^2 = m^2 + \sigma R \ , \tag{34}$$

with $\sigma \equiv \frac{2}{3}\,a - b$. Using (33), the action (11) takes the form

$$S_{sd\lambda} = \int \frac{d^3x}{2}\sqrt{-g}(m\,\varepsilon^{\mu\nu\lambda}h_\mu{}^\alpha\nabla_\nu h_{\lambda\alpha} - M^2\,(h_{\mu\nu}h^{\nu\mu} - h^2))\ , \tag{35}$$

and with the help of (22) and (33), the object $\mathcal{N}^{ij}$ becomes

$$\mathcal{N}^{ij} \equiv \left(\frac{6M^4 - Rm^2}{m}\right)\varepsilon^{ij}\ . \tag{36}$$

The consistence relation (30) is now

$$6M^4 - Rm^2 \neq 0\ . \tag{37}$$

Considering (34) we can think about this relation as a restriction on m^2 in terms of scalar curvature and σ. This means

$$m^2 \neq m_\pm{}^2 \equiv \frac{R}{12}\left(1 - 12\sigma \pm \sqrt{1-24\sigma}\right)\ , \tag{38}$$

showing the existence of some forbidden mass values in order to have consistency, which represents a well known fact in context of higher spin theories[2].

Now, lagrangian constraints are revisited, this time in dS/AdS space. The primary nine, (13) are

$$\Phi^{(1)\mu\alpha} \equiv m\,\varepsilon^{\mu\nu\lambda}\nabla_\nu h_\lambda{}^\alpha + M^2\,(g^{\mu\alpha}h - h^{\alpha\mu}) \approx 0\ . \tag{39}$$

Next, secondary constraints (14)

$$\Phi^{(2)\alpha} \approx M^2\,(\nabla^\alpha h - \nabla_\mu h^{\alpha\mu}) + \frac{mR}{6}\,\varepsilon^{\alpha\sigma\lambda}h_{\sigma\lambda} \approx 0\ , \tag{40}$$

which are written as $\Phi^{(2)\alpha} \approx \nabla_\mu\Phi^{(1)\mu\alpha} - \frac{M^2}{m}\,\varepsilon^{\alpha\sigma\lambda}\Phi^{(1)}{}_{\mu\alpha} = \left(\frac{m^2R-6M^4}{6m}\right)\varepsilon^{\alpha\sigma\lambda}h_{\sigma\lambda} \approx 0$. So, the symmetry property for the $h_{\mu\nu}$ field, in virtue of (37), is gained.

Preservation of $\Phi^{(2)\alpha} \approx 0$ provides three more constraints

$$\Phi^{(3)\alpha} \approx \left(\frac{m^2R - 6M^4}{6m}\right)\varepsilon^{\alpha\sigma\lambda}\nabla_0 h_{\sigma\lambda} \approx 0\ . \tag{41}$$

Its temporal component, $\Phi^{(3)0} \approx 0$ is expressed as

$$\Phi^{(3)0} \approx \nabla_\mu \Phi^{(2)\mu} + \left(\frac{6M^4 - m^2 R}{6m^2}\right) \Phi^{(1)\mu}{}_\mu = \frac{M^2}{3m^2}(6M^4 - m^2 R)h \approx 0 \ . \tag{42}$$

It says that the spin 2 field is traceless (obviously if $M^2 \neq 0$). The last constraint arise from the preservation of $\Phi^{(3)0} \approx 0$

$$\Phi^{(4)} \equiv \nabla_0 \Phi^{(3)0} \approx \frac{M^2}{3m^2}(6M^4 - m^2 R)\nabla_0 h \approx 0 \ . \tag{43}$$

We observe that traceless and transverse properties for a consistent description of the selfdual field, demands the additional condition

$$M^2 \neq 0 \ , \tag{44}$$

as a consequence of equations (40), (42) and (43). If one relax this restriction on M^2 (i.e., $M^2 = 0$), the lagrangian system will not furnish the expected number of degrees of freedom.

Imposing (44) we can construct a quadratical Klein-Gordon-like field equation for $h^{(s)Tt}{}_{\mu\nu}$, as follows

$$(\Box - \frac{M^4}{m^2} + \frac{R}{2}) h^{(s)Tt}{}_{\mu\nu} = 0 \ . \tag{45}$$

with $\Box \equiv \nabla_\alpha \nabla^\alpha$. This equation is clearly hyperbolic and causal, because we can rewrite it in the form

$$(\mathcal{M}^{\beta\sigma}{}_{\rho\alpha})^{\mu\nu} \nabla_\mu \nabla_\nu h^{(s)Tt}{}_{\beta\sigma} + ... = 0 \ . \tag{46}$$

where $(\mathcal{M}^{\beta\sigma}{}_{\rho\alpha})^{\mu\nu} = g^{\mu\nu}\delta^{\beta\sigma}{}_{\rho\alpha}$ and $\delta^{\beta\sigma}{}_{\rho\alpha} \equiv \frac{1}{2}(\delta^\beta{}_\rho \delta^\sigma{}_\alpha + \delta^\sigma{}_\rho \delta^\beta{}_\alpha)$. Then, with the help of the three-vectors n_μ, we define the characteristic matrix

$$\mathcal{M}^{\beta\sigma}{}_{\rho\alpha}(n) = \delta^{\beta\sigma}{}_{\rho\alpha}\, n^2 \ , \tag{47}$$

whose characteristic equation is

$$det(\mathcal{M}) = (n^2)^6 = 0 \ , \tag{48}$$

which has a null vector solution.

As dS/AdS space are conformally flat, its light cones are equivalent to those of Minkowski (i.e., they are related through a Weyl map), and we can write $n^2 = 0$ in a locally Weyl-flat frame (using the fact that the conformal transformation for metric is $g_{\mu\nu} = \Omega^2 \eta_{\mu\nu}$), as follows

$$- (n_0)^2 + n_i n_i = 0 \ , \tag{49}$$

which describes an hyperbolic (n_0 is real) and causal ($n^2 = 0$ implies there is not time-like three-vectors) propagation.

On the other hand, the selfdual massive theory studied holds forbidden mass values in dS/AdS spaces because of (44) (i.e., $m^2 \neq -\sigma R$). These values can be resumed as follows

R	σ	$forbidden\ m$
$> 0\,(AdS)$	$0 \leq \sigma \leq \frac{1}{24}$	$m_{\pm}$
$> 0\,(AdS)$	< 0	$\sqrt{-\sigma R}$
$< 0\,(dS)$	> 0	$\sqrt{-\sigma R}$

We note that, in the study of the selfdual massive spin 2 field theory coupled with gravity a well known fact is verified: inside a possible set of solutions, those of constant curvature spaces respect the number of degrees of freedom and causality. However, in contrast to other type of spin 2 theories[11], the selfdual massive one does not have massless limit, and $M^2 \neq 0$ is demanded in order to guarantee equivalence between constraint system and symmetry, traceless and transverse properties of selfdual massive field with an hyperbolic and causal equation provided.

There are other issues related with the condition $M^2 \neq 0$. On one hand, this condition, in a dS/AdS background mantains the selfdual lagrangian, (35), conformally variant due to the non null trace of the energy-momentun associated with the selfdual field, $T^{\mu}{}_{\mu} = -\frac{M^2}{2}\, h^{(s)Tt}{}_{\mu\nu} h^{(s)Tt\mu\nu}$.

Moreover, the critical value $M^2 = 0$ reveals the existence of an expected discontinuity in the degrees of freedoms count, because it gives rise to a non consistent set of lagrangian constraints, which is associated with the nature of the quadratical terms in the action, and not to the features of the gravitational interaction. This kind of discontinuity can be illustrated from in the flat space model, as follows. Let us consider the two parameter action

$$S_{m_1,m_2} = \int d^3x(\frac{m_1}{2}\,\epsilon^{\mu\nu\lambda}h_\mu{}^\alpha\partial_\nu h_{\lambda\alpha} - \frac{{m_2}^2}{2}(h_{\mu\nu}h^{\nu\mu} - h^2))\ , \tag{50}$$

which reproduces the selfdual massive model when we choose $m_1 = m_2 = m$. Particularly, when $m_2 = 0$, the action (50) describes a model with no degrees of freedom (in fact, the reduced action becomes null identically). However, if $m_2 \neq 0$ is considered during the procedure that lead us to the reduced action, one can arrive to the expected relation: ${S_{m_1,m_2}}^* = \int d^3x\{P\dot{Q} - \frac{1}{2}P^2 + \frac{1}{2}\,Q(\Delta - \mathbb{M}^2)Q\}$, whith $\mathbb{M} \equiv \frac{{m_2}^2}{m_1}$, $P \equiv \sqrt{2}\,m_2\Delta h^{(s)TL}$ and $Q \equiv \sqrt{2}\,\frac{m_1}{m_2}\Delta h^{(s)TT}$, which is a singular function at $m_2 = 0$, saying that the model (50) does not have a well defined limit at $m_2 = 0$.

In an analogous way, a discontinuity does appear in the selfdual massive model when we consider a dS/AdS background, (35) at the critical value $M^2 = 0$. In fact, this behavior is manifest if we observe that equation (39) is now gauge invariant under

$$\delta h^{(s)Tt}{}_{\mu\nu} = (\nabla_\mu\nabla_\nu - \frac{R}{6}\,g_{\mu\nu})\omega(x)\ , \tag{51}$$

which says that the only degree of freedom due to $h^{(s)Tt}{}_{\mu\nu}$, can be gauged away and the theory with $M^2 = 0$ does not propagating degrees of freedom as in the flat case.

If in the context of a curved space-time, we want to realize a procedure in order to obtain a reduced action for selfdual massive spin 2 theory, and then description of the only propagated degree of freedom through a field like ${h^{(s)Tt}{}_{\mu\nu}}^\pm$. In the flat case it can be seen that the symbol "±" is associated with a propagation of spin ±2[21]. In a curved space-time this "flat" procedure will find serious obstacles. Essentially, this is related with the problem of the Fourier transform in curved spaces[22] and the definition of arbitrary powers of D'Alembertian, and as a consecuence of the obscure business to obtain projectors.

However, we can say something following a covariant procedure in order to obtain the reduced action, starting with the a symmetric-antisymmetric decomposition

$$h_{\mu\nu} \equiv h^{(s)}{}_{\mu\nu} + \varepsilon_{\mu\nu\lambda} V^{\lambda} \ . \tag{52}$$

Using this in (35), conduce us to

$$S_{sd\lambda} = \int d^3x \sqrt{-g} \Big(\frac{m}{2} \varepsilon^{\mu\nu\sigma} g^{\beta\alpha} h^{(s)}{}_{\mu\beta} \nabla_\nu h^{(s)}{}_{\sigma\alpha} - \frac{M^2}{2} (h^{(s)}{}_{\mu\nu} h^{(s)\mu\nu} - {h^{(s)}}^2) + \\ + m V^\mu (\nabla_\mu h^{(s)} - \nabla_\nu h^{(s)}{}_\mu{}^\nu) - \frac{m}{2} \varepsilon^{\mu\nu\sigma} V_\mu \nabla_\nu V_\sigma - M^2 V_\mu V^\mu \Big) \ , \tag{53}$$

and the field equations

$$m\varepsilon^{\mu\nu\lambda} \nabla_\mu h^{(s)}{}_\nu{}^\alpha + m\varepsilon^{\mu\nu\alpha} \nabla_\mu h^{(s)}{}_\nu{}^\lambda - 2M^2 h^{(s)\lambda\alpha} + 2M^2 g^{\lambda\alpha} h^{(s)} + \\ -2m g^{\lambda\alpha} \nabla_\mu V^\mu + m(\nabla^\lambda V^\alpha + \nabla^\alpha V^\lambda) = 0 \ , \tag{54}$$

$$m\varepsilon^{\mu\nu\lambda} \nabla_\mu V_\nu + 2M^2 V^\lambda - m\nabla^\lambda h^{(s)} + m\nabla_\mu h^{(s)\mu\lambda} = 0 \ . \tag{55}$$

The trace and divergence of (54) give

$$M^2 h^{(s)} - m\nabla_\mu V^\mu = 0 \ , \tag{56}$$

$$m\varepsilon^{\mu\nu\lambda} \nabla_\nu \mathcal{H}_\lambda - 2M^2 \mathcal{H}^\mu + 2M^2 \nabla^\mu h^{(s)} + m\Box V^\mu + \\ - m\nabla^\mu \nabla_\alpha V^\alpha - \frac{mR}{3} V^\mu = 0 \ , \tag{57}$$

with the notation $\mathcal{H}_\lambda \equiv \nabla_\alpha h^{(s)}{}_\lambda{}^\alpha$. Using (57) in (55), we get

$$(Rm^2 - 6M^4) V_\sigma = 0. \ , \tag{58}$$

Taking into account the restriction (37), we get $V_\sigma = 0$. This last relation with (56) gives the suplementary $h^{(s)} = 0$, and equation (55) assure $\mathcal{H}_\lambda \equiv \nabla_\alpha h^{(s)}{}_\lambda{}^\alpha = 0$. Then, it is confirmed that in constant curvature spaces the selfdual massive spin 2 theory is described by a symmetric-transverse-traceless field, $h^{(s)Tt}{}_{\mu\nu}$, and the reduced action will take the form

$$S_{sd\lambda}{}^{(2)*} = \int d^3x\,\sqrt{-g}\,\big(\,\frac{m}{2}\,\varepsilon^{\mu\nu\sigma}h^{(s)Tt}{}_{\mu}{}^{\alpha}\nabla_\nu h^{(s)Tt}{}_{\sigma\alpha} - \frac{M^2}{2}\,h^{(s)Tt}{}_{\mu\nu}h^{(s)Tt\mu\nu}\big)\ , \qquad (59)$$

which the equations of motion

$$m\,\varepsilon^{\sigma\mu\nu}\nabla_\mu h^{(s)Tt}{}_{\nu}{}^{\beta} - M^2 h^{(s)Tt\sigma\beta} = 0\ . \qquad (60)$$

From this, the causal propagation (45) is obtained.

At a point p of the manifold $\mathcal{M}$, it can be attached a tangent space, $T_p(\mathcal{M})$ with locally coordinates ξ^a provided. So, in this reference the hyperbolic-causal equation is

$$\big(\Box_{(\xi)} - \frac{M^4}{m^2} + \frac{R}{2}\big)h^{(s)Tt}{}_{ab}(\xi) = 0\ , \qquad (61)$$

with $\Box_{(\xi)} \equiv \partial^a\partial_a$. Next, we define the locally "+" and "−" parts of $h^{(s)Tt}{}_{ab}(\xi)$ in the way

$$h^{(s)Tt}{}_{ab}{}^{\pm} \equiv \frac{1}{2}\bigg(\frac{\delta^d{}_a\delta^c{}_b}{q} \pm \delta^d{}_a\epsilon_b{}^{rc}\frac{\partial_r}{\Box_{(\xi)}{}^{\frac{1}{2}}}\bigg)h^{(s)Tt}{}_{dc}\ , \qquad (62)$$

where the parameter $q \equiv \sqrt{1-\frac{Rm^2}{2M^4}}$. Then, with the local on-shell relation (61), it can be obtained that

$$\big(\Box_{(\xi)} - \frac{M^4}{m^2} + \frac{R}{2}\big)h^{(s)Tt}{}_{ab}{}^{\pm}(\xi) = 0\ , \qquad (63)$$

$$h^{(s)Tt}{}_{ab}{}^{\mp}(\xi) = 0\ , \qquad (64)$$

saying that the only degree of freedom locally propagated is described through $h^{(s)Tt}{}_{ab}{}^{+}$ ($h^{(s)Tt}{}_{ab}{}^{-}$), if the spin is $+2(-2)$. It can be observed that expression (62) can be rewritten as $h^{(s)Tt}{}_{ab}{}^{\pm} \equiv P^{\pm dc}{}_{ab}h^{(s)Tt}{}_{dc}$, where

$$P^{\pm dc}{}_{ab} \equiv \frac{1}{4}\bigg(\frac{1}{q}\big(\delta^d{}_a\delta^c{}_b + \delta^c{}_a\delta^d{}_b\big) \pm \big(\delta^d{}_a\epsilon_b{}^{rc} + \delta^c{}_a\epsilon_b{}^{rd}\big)\,\frac{\partial_r}{\Box_{(\xi)}{}^{\frac{1}{2}}}\bigg)\ , \qquad (65)$$

is not a projector (i.e., $P^{\pm dc}{}_{ab}P^{\pm ab}{}_{ef} \neq P^{\pm dc}{}_{ef}$), because $q \neq 1$.

Finally, we examine the one-particle exchange amplitude which describes the interaction between sources. This starts with the selfdual massive spin 2 action in the form (53), but now minimally coupled with an external, symmetric, conserved source ${T^{(s)}}_{\mu\nu}(x)$ (i.e., $\nabla^\mu {T^{(s)}}_{\mu\nu}(x) = 0$) as follows

$$S_{sd\lambda s} = \int d^3x \sqrt{-g}\Big(\frac{m}{2}\, \varepsilon^{\mu\nu\sigma} g^{\beta\alpha} {h^{(s)}}_{\mu\beta} \nabla_\nu {h^{(s)}}_{\sigma\alpha} - \frac{M^2}{2}\, ({h^{(s)}}_{\mu\nu} h^{(s)\mu\nu} - {h^{(s)}}^2) + \\ + mV^\mu(\nabla_\mu h^{(s)} - \nabla_\nu {{h^{(s)}}_\mu}^\nu) - \frac{m}{2}\, \varepsilon^{\mu\nu\sigma} V_\mu \nabla_\nu V_\sigma - M^2 V_\mu V^\mu + \kappa {h^{(s)}}_{\mu\nu} T^{(s)\mu\nu}\Big) \ , \tag{66}$$

where κ is a coupling parameter.

The field equations are emerging from (66) are

$$m\varepsilon^{\mu\nu\lambda} \nabla_\mu {{h^{(s)}}_\nu}^\alpha + m\varepsilon^{\mu\nu\alpha} \nabla_\mu {{h^{(s)}}_\nu}^\lambda - 2M^2 h^{(s)\lambda\alpha} + 2M^2 g^{\lambda\alpha} h^{(s)} + \\ -2mg^{\lambda\alpha} \nabla_\mu V^\mu + m(\nabla^\lambda V^\alpha + \nabla^\alpha V^\lambda) = -2\kappa T^{(s)\lambda\alpha} \ , \tag{67}$$

$$m\varepsilon^{\mu\nu\lambda} \nabla_\mu V_\nu + 2M^2 V^\lambda - m\nabla^\lambda h^{(s)} + m\nabla_\mu h^{(s)\mu\lambda} = 0 \ . \tag{68}$$

Divergence and trace of (67) give

$$\varepsilon^{\mu\nu\lambda} \nabla_\nu \nabla_\alpha {{h^{(s)}}_\lambda}^\alpha - \frac{2M^2}{m} \nabla_\alpha h^{(s)\mu\alpha} + \frac{2M^2}{m} \nabla^\mu h^{(s)} - 2m\nabla^\mu \nabla_\lambda V^\lambda + \\ + \nabla_\lambda \nabla^\mu V^\lambda + \Box V^\mu = 0 \ , \tag{69}$$

$$2M^2 h^{(s)} - 2m\nabla_\mu V^\mu = -\kappa T^{(s)} \ . \tag{70}$$

Curl of (68) is

$$-\, {\varepsilon_\sigma}^{\rho\mu} \nabla_\rho \nabla_\nu {{h^{(s)}}_\mu}^\nu - \Box V_\sigma + \nabla_\lambda \nabla_\sigma V^\lambda - \frac{2M^2}{m} \varepsilon_{\sigma\rho\mu} \nabla^\rho V^\mu = 0 \ , \tag{71}$$

and with the help of (69), conduce us to

$$(m^2 R - 6M^4) V_\sigma = 0 \ , \tag{72}$$

which again says that $V_\sigma = 0$. Then we can rewrite (67), (68) and (70) as follows

$$m\varepsilon^{\mu\nu\lambda}\nabla_\mu h^{(s)}{}_\nu{}^\alpha + m\varepsilon^{\mu\nu\alpha}\nabla_\mu h^{(s)}{}_\nu{}^\lambda - 2M^2 h^{(s)\lambda\alpha} + 2M^2 g^{\lambda\alpha} h^{(s)} = -2\kappa T^{(s)\lambda\alpha} \ , \tag{73}$$

$$\nabla^\lambda h^{(s)} - \nabla_\mu h^{(s)\mu\lambda} = 0 \ , \tag{74}$$

$$2M^2 h^{(s)} = -\kappa T^{(s)} \ . \tag{75}$$

For the computation of the exchange amplitude we need the decomposition

$$h^{(s)}{}_{\mu\nu} = h^{(s)Tt}{}_{\mu\nu} + \nabla_\mu a^T{}_\nu + \nabla_\nu a^T{}_\mu + \nabla_\mu \nabla_\nu \phi + g_{\mu\nu}\psi \ , \tag{76}$$

where $\nabla^\mu a^T{}_\mu = 0$. The following relations arise from (76)

$$h^{(s)} = \Box\phi + 3\psi \ , \tag{77}$$

$$(-\Box + \frac{R}{3})a^T{}_\mu + \frac{R}{3}\nabla_\mu\phi + 2\nabla_\mu\psi = 0 \ , \tag{78}$$

where the last one is obtained with the help of (74). Divergence of (78) provides $R\Box\phi + 6\Box\psi = 0$, and using this in (77) with (75), we get

$$(\Box - \frac{R}{2})\psi = \frac{R\kappa}{12M^2} T^{(s)} \ . \tag{79}$$

Now we need to write down $h^{(s)Tt}{}_{\mu\nu}$ in terms of the source. The Tt part of (73) is

$$m\varepsilon^{\mu\nu\lambda}\nabla_\nu h^{(s)Tt}{}_\lambda{}^\alpha - M^2 h^{(s)Tt\mu\alpha} = -\kappa T^{(s)Tt\mu\alpha} \ , \tag{80}$$

from which we obtain the hyperbolic-causal equation for $h^{(s)Tt}{}_{\mu\nu}$

$$\big(\Delta^{(2)} + \frac{M^4}{m^2} + \frac{R}{2}\big) h^{(s)Tt\mu\alpha} = \frac{\kappa M^2}{m^2} T^{(s)Tt\mu\alpha} + \frac{\kappa}{m}\,\varepsilon^{\mu\rho\sigma}\nabla_\rho T^{(s)Tt}{}_\sigma{}^\alpha \ , \tag{81}$$

where $\Delta^{(2)}$ is the Lichnerowicz operator which obeys the following properties[23]

$$\Delta^{(0)}\phi = -\Box\phi \ , \tag{82}$$

$$\nabla^\mu \Delta^{(1)} V_\mu = \Delta^{(0)} \nabla^\mu V_\mu \ , \tag{83}$$

$$\Delta^{(2)}\nabla_{(\mu}V_{\nu)} = \nabla_{(\mu}\Delta^{(1)}V_{\nu)} \ , \tag{84}$$

$$\nabla^{\mu}\Delta^{(2)}h^{(s)}{}_{\mu\nu} = \Delta^{(1)}\nabla^{\mu}h^{(s)}{}_{\mu\nu} \ , \tag{85}$$

$$\Delta^{(2)}(g_{\mu\nu}\phi) = g_{\mu\nu}\Delta^{(0)}\phi \ , \tag{86}$$

and $T^{(s)Tt}{}_{\mu\nu}$ is given by

$$T^{(s)Tt}{}_{\mu\nu} = T^{(s)}{}_{\mu\nu} - \frac{g_{\mu\nu}}{2}T^{(s)} + \frac{1}{2}(\nabla_{\mu}\nabla_{\nu} - \frac{R}{6}g_{\mu\nu})(\Box - \frac{R}{2})^{-1}T^{(s)} \ . \tag{87}$$

The exchange amplitude between two covariant conserved sources is $A = \int d^3x\sqrt{-g}\mathcal{A}$, where $\mathcal{A} \equiv T'^{(s)}{}_{\mu\nu}h^{(s)\mu\nu}$. Up to boundary terms we can write A using

$$\mathcal{A} = T'^{(s)}{}_{\mu\nu}h^{(s)Tt\mu\nu} + T'^{(s)}\psi \ . \tag{88}$$

Considering (79), (81) and (87), we obtain

$$\begin{aligned} \frac{\mathcal{A}}{\kappa} = & \frac{M^2}{m^2}\ T'^{(s)}{}_{\alpha\beta}(\Delta^{(2)} + \mu^2)^{-1}T^{(s)\alpha\beta} - \frac{M^2}{2m^2}\ T'^{(s)}(-\Box + \mu^2)^{-1}T^{(s)} + \\ & + \frac{2}{m}\ T'^{(s)}{}_{\alpha\beta}(\Delta^{(2)} + \mu^2)^{-1}\varepsilon^{(\alpha\rho\sigma}\nabla_{\rho}T^{(s)Tt\beta)}{}_{\sigma} - \frac{R}{12m^2}\ T'^{(s)}(-\Box + \frac{R}{2})^{-1}T^{(s)} + \\ & + \frac{M^2R}{12m^2}\ T'^{(s)}(-\Box + \mu^2)^{-1}(-\Box + \frac{R}{2})^{-1}T^{(s)} \ , \end{aligned} \tag{89}$$

where $\mu^2 \equiv \frac{M^4}{m^2} + \frac{R}{2}$.

In the flat limit, looking at the first three terms in (89), two of them will be proportional to $\frac{M^2}{m^2}$ and the other to $\frac{2}{m}$. They correspond to the amplitude of a massive selfdual massive spin 2 in 2+1 dimensions. For the remaining terms they give a cosmological contribution which disappears in the flat limit. In the curved case it can be observed that these last terms have an unphysical pole at $\Box = \frac{R}{2}$ which do not propagate (i.e., the residue in the amplitude is $\frac{M^2R}{12m^2}(-\frac{R}{2} + \mu^2)^{-1} - \frac{R}{12M^2} = 0$, whatever the sign of the cosmological constant). On the other hand, the physical pole $\Box = \mu^2$ has the residue

$$\mathcal{R}_{(\Box=\mu^2)} = -\kappa\frac{M^2}{2m^2}\big(1 - \frac{\lambda m^2}{M^4}\big) \ , \tag{90}$$

which is clearly non null in an AdS space-time.

As a concluding remark, the selfdual massive spin 2 model in dS/AdS background exhibits frobidden mass values in order to guarantee consistency, and they are given by (37) and (44)

$$6M^4 - Rm^2 \neq 0 \ , \tag{91}$$

$$M^2 \neq 0 \ , \tag{92}$$

with $M^2 = m^2 + \sigma R$. Both contain information about the background and they match in the flat space-time limit with the consistence condition of the selfdual massive model: $m \neq 0$. Moreover, M^2 does appear as a quadratical power of a "mass" in action (35), this can be thought as the curvilinear version of the two parameters flat action, (50)(which contains the flat selfdual massive spin 2 model). So, the presence of $M^2 \neq 0$ in the action (35), guarantees a conformally variant selfdual massive model in dS/AdS, matching with the same situation in flat theory. However, one can distinguish between dS or AdS because the sign of the residue, $\mathcal{R}_{(\Box=\mu^2)}$ is sensitive when $\lambda > 0$ (dS).

Aknowlegments

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