POLITECNICO DI TORINO

Department of Environment, Land and Infrastructure Engineering

Master of Science in Petroleum Engineering

COMPARISON STUDY BETWEEN ANALYTICAL AND NUMERICAL AQUIFER MODELS BY RESERVOIR SIMULATION

Supervisor:

Prof. Dario Viberti

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Student Name:

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Acknowledgement

Firstly, this thesis is dedicated to God and the Virgen María Rosa Mística, for every new day they are continually giving me and thus a new opportunity to be better in all aspects of my life.

Quiero dedicar esta tesis en primer lugar a Dios y a la Virgen María Rosa Mística por estar presente en mi vida a cada segundo y a cada instante, por acompañarme en este largo recorrido y por cada nuevo amanecer que me siguen regalando, que no es más que una nueva oportunidad para ser mejor cada día. En segundo lugar, a mis padres Wilfredo Martínez, Sally Acevedo y a mi hermana Paola Martínez, quienes me han dado la fuerza y fortaleza en los momentos difíciles para alcanzar este logro. Gracias a mi mamá por siempre estar a mi lado cuando la necesite, en los momentos buenos y amargos, por su dedicación y constancia, su esfuerzo y esmero, no tendré palabras nunca que me alcancen para agradecerte y devolverte todo lo que me has dado. Gracias a mi papá, por sus consejos y regaños, por su ardua labor y sacrificios, gracias infinitas, porque sin ustedes mi vida no estaría completa.

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Abstract

This thesis established a comparison between aquifer models to better understand whether they can represent an aquifer in the same way under consistent conditions. The aquifer models selected correspond to Fetkovich, Carter-Tarcy and Numerical aquifer models, basically due to their ease applicability and availability in the simulation tool ECLIPSE 100 (a Schlumberger simulator tool). While for single wells or other small systems simple cells analysis may be adequate, large complex systems demand a much more sophisticated approach to predict the response of a large complicated production system accurately and to examine alternative operational scenarios efficiently. Consequently, the aquifer system has been extremely simplified by one singular cell in the simulation tool, in order to understand in the best way the performance of the models. Furthermore, a sensitivity analysis was performed to analyze the response of the analytical models when aquifer parameters were changed. The modifications in aquifer parameters correspond to alterations on permeability and net pay values of the aquifer. Accordingly, different comparatives cases were carried out following a main production constraint (constant bottom-hole pressure and constant flow rate), in order to identify important or drastically changes between the models. Finally, a sensitivity analysis was done to evaluate the behavior of both analytical models selected and a numerical aquifer model, maintaining a consistent production constraint and aquifer properties, as well as.

The results of this investigation exposed the important influence of flow regimes into the execution of the models, due to the pronounced effect that permeability values have over their responses, especially on Fetkovich aquifer model outputs. Moreover, changes in the net pay values did not represent a major factor for changing the behavior of the models, while representing the same aquifer system. Additionally, congruent results between analytical and numerical aquifer model responses were not found, due to principally their theoretical bases, and thus their different equations implemented to characterize an aquifer body.

Keywords: Aquifer, Analytical, Numerical, Reservoir, Simulation.

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1. INTRODUCTION

Hydrocarbon reservoirs are often surrounded by an aquifer. If the aquifer is big enough, during the reservoir exploitation, a flow of water (water influx) from the aquifer to the reservoir is induced by the pressure drop generated due to the production of the hydrocarbon.

The main target of reservoir simulation consists in generating a 3D numerical model able to reproduce the historical dynamic behavior of the reservoir and, when calibrated, to provide reliable production forecasts. However, the calibration of a reservoir numerical model requires an high (and sometimes huge) number of simulations. According to the dimension of the problem (in terms of number of grid cells), each simulation could take from few minutes to hours. In order to reduce the number of cells in the model and the simulation time, the presence of the aquifer is simulated adopting analytical solutions (analytical aquifer) and/or simplified numerical sub-domains of the grid (numerical aquifer).

Several analytical aquifer models have been implemented through the history, the main are: Van Everdingen and Hurst (1949), Carter-Tracy (1960), Fetkovich (1971), and Allard & Chen. However, it is necessary to understand the differences between these models in order to better match the behavior of real aquifers.

The target of the thesis was a throughout comparison between some of the aquifer models adopted in reservoir simulation. First of all, by means of the calibration of the models, afterwards, establishing the circumstances that must be reached in order to observe discrepancies between them. The simulator used during the study is Eclipse 100 (a Schlumberger simulator tool). This investigation has selected Fetkovich and Carter-Tracy aquifers models as two analytical models available in the simulator tool, but also a numerical aquifer model. While for single wells or other small systems simple cells analysis may be adequate, large complex systems demand a much more sophisticated approach to predict the response of a large complicated production system accurately and to examine alternative operational scenarios efficiently. Consequently, the aquifer system has been extremely simplified by one singular cell in the simulation tool, in order to understand in the best way the performance of the models.

In order to verify when the models start to behave in a different way between them, it has been carried out a sensitivity analysis by means of a number of tests changing the main parameters of an aquifer, as for instance, permeability and net pay values.

Finally, a contrast between the correspondence in representing the same aquifer system through analytical and numerical aquifer models was developed, analyzing a simplified model composed of 2 cells in the simulation tool.

2. LITERATURE REVIEW

Aquifer models are based on mathematical representations that simulate and forecast aquifer performance. They are used to predict the cumulative water influx history that is a fundamental factor to have in consideration while the production of a field. Hence, it is also important to do a cognitive bibliography revision of all the theories that can be applied to study the behavior of an aquifer and the methods that can be used for calculating water influx into a reservoir.

Among the classic models used in the evaluation of aquifer properties, the Schilthuis steady-state model was proposed by **Schilthuis (1936)** as a model for an aquifer that was flowing under steady-state flow regime, and where the flow behavior could be described by Darcy's equation. Furthermore, problems associated with Schilthuis steady-state model consisted in the increment of the aquifer drainage radius as the time increases, due to the fact that water is drained from the aquifer in time. Moreover, the most realistic method for calculating water influx rate was proposed by **Van Everdingen & Hurst (1949),** who applied the Laplace transformation in order to solve the diffusivity equation. It founded the general mathematical equation that was designed to model the transient flow behavior in reservoirs or aquifers, considering it for the reservoir-aquifer system, and taking as boundary condition either a constant pressure or a constant rate in the boundary. A stronger computational capacities were required particularly in the evaluation of the Everdingen & Hurst model due to the execution of Duhamel superposition principle. Moreover, another recognized water influx model was Hurst Simplified Method **(Hurst, 1956),** who formulated theories for edge-water drive in linear and radial cases, using basically material balance equations and enforcing the diffusivity equation's solution in Laplace space (Van Everdingen and Hurst, 1949). Applying Laplace space allowed to this so called "simplified" model, to find a shortened expression for drawdown period as a clear function of production rate and time. Hurst's theories developed an easily model to adapt in geothermal reservoir systems. **Olsen (1964)** re-derived the Hurst linear model for geothermal applications. **Miller (1962)** centered his investigation in the performance of a closed outer boundary and infinite aquifer, creating and analyzing different trends for each aquifer size. **Nabor and Barham (1964)** extended Miller's (1962) calculations and developed a single log-log type curve analysis applicable to any aquifer size. Both, **Miller (1962)** and **Nabor and Barham (1964)** included analysis related to the case for constant pressure outer boundary.

Additionally, **Mueller (1962)** studied the problematic for non-homogeneous aquifer responses by means of finite-difference techniques. **Mueller (1962)** observed direct disparities of thickness, permeability or porosity-compressibility product with distance. **Bowman and Crawford (1962)** developed a method to better focus the transient pressure distribution in linear semi-infinite water-drive systems, analyzing different rock and fluid characteristics in each zone.

Fanchi (1985) matched the Van Everdingen and Hurst tabulated values of the dimensionless pressure as a function of dimensionless time and dimensionless radius. In order to develop the polynomial form of the solution giving W_D as a function of t_D, for a range of ratios of the aquifer to reservoir radius $r_{eD} = r_e/r_D$, the Van Everdingen & Hurst tabulated value were matched. Recently**, Ambastha and Ramey (1987)** obtained a suite of analytical response functions for non-homogeneous aquifers. Their results compared well with **Mueller's (1962)** results.

Moreover, a report was submitted to the Department of petroleum engineering at Stanford University called "Compressibility effects in modeling two-phase liquid dominated geothermal reservoirs" by **Brock, D. C. 1986**. It was studied the use of the Hurst Simplified Model to history match the drawdown behavior of the liquid dominated geothermal reservoirs, concluding that the Hurst Simplified Method history match yields useful reservoir parameters (c and kh) as well as, a model useful in prediction.

Nonetheless, due to the necessity to further simplify water influx calculations, Fetkovich **(M.J.Fetkovich, 1971)** proposed a pseudosteady-state aquifer model that used a productivity index and an aquifer material balance in order to represent the compressibility of the system. Fetkovich's model removed the necessity to implement the superposition principle proposed and applied for **Van Everdingen & Hurst (1949),** making Fetkovich aquifer model a more easy alternative to study water influx. However, Fetkovich presented some inconvenient trying to calculate water influx under transient time, since Fetkovich aquifer model neglected the early transient time, thus producing a minor performance, in comparison with other models, when calculations were done for aquifer water influx. Additionally, in a very similar way with respect to the fluid flow coming from a reservoir to a well, Fetkovich employed an inflow equation to exemplify the water influx from the aquifer to the reservoir. Although Fetkovich model was defined for finite aquifers, it can be extended to infinite-acting aquifers, which required approximately constant values of the water influx rate to pressure drop ratio during the productive life of the reservoir.

Meanwhile, **Carter and Tracy (1960)** proposed an aquifer model trying to further abridge the complexity of **Van Everdingen & Hurst (1949),** projecting several calculations techniques that did not require superposition principle, and thus making direct water influx calculations. Differences between **Carter and Tracy (1960)** and **Van Everdingen & Hurst (1949)** lies to the fact that the Carter and Tracy model assumed constant water influx rates over each finite time interval. Using the Carter-Tracy technique, the cumulative water influx at any time, t_n, could be calculated directly from the previous value obtained at t_{n-1}. For an infinite-acting aquifer, **Edwardson et al. (1962)** developed an approximation of p_D as a function of dimensionless time t_D and for a given dimensionless radius r_D.

Furthermore, along the history, other theories have been developed, as for instance, **Allard & Chen (1984),** who established a numerical simulation for bottom-water drive systems to understand better the situations in which the ratio of reservoir thickness to reservoir radius increases not accurately modelled by edge-water models. Beside this, **The Leung models (1986),** were also considered as a strong alternative to study aquifer-resevoir systems, since a pseudosteady-state model and a modified pseudosteady-state model were established. To better understand finite aquifers under pseudosteady-state regimes a pseudosteady-state model was proposed, while the modified pseudosteady-state was suggested for a better representation of the system, since it took into account the effects of transient time period on the performance of the model. Leung models were also characterized for an ease way of application, since as other models, computational efforts related to the superposition principle were removed when the pressure at the boundary (aquifer-reservoir) varies with time.

The main goal of this investigation has been focus to the comparison between aquifer models, including analytical and numerical models, in order to find possible deviations in the performance of the models while they are subjected to an aquifer system. Accordingly, it was followed their rigorous statements of theory, when they were implemented with a reservoir simulator.

Nevertheless, several and more recent studies have been carried out in order to obtain further information about the veracity and accuracy of the most common models to study the behavior of the aquifers. An example of this, a comparative study **(Marques, J. B. & Trevisan, O. V. Buenos Aires, Argentina, 15-18 April 2007)** done with the total influx of water by the aquifer performance as a function of time, taking as base case the model proposed by Van Everdingen & Hurst (1958), considering it as the best in terms of solution of the diffusive equation. All analyses were done in computer spreadsheets using the equations proposed by the authors of each model.

Carter & Tracy (1986) presented models taking into account the effects of the aquifer transient period, making them the most effective models in terms of speed and simplicity, compared to that of Van Everdingen & Hurst.

On the another hand, many studies to understand and verify the Van Everdingen-Hurst model have been carried out, as an example, a study was presented in order to determine water influx using Van Everdingen-Hurst model **(Oloro J., & Erhimudia U., Delta State University, July, Nigeria, 2011),** comparing the final results with the company model for calculating also water influx. The main goal of this comparison was to determine the suitability of the model in estimating water influx in Niger Delta using Van Everdingen-Hurst unsteady-state model. From the test carried out was concluded that there was significant difference between both models, while for exact solution Van Everdingen-Hurst was suitable. However, for easy and approximate solution, Company (HURST-VANEVERDINGEN-ODEH) model was preferred. At the end was proposed to compare the results also with Cater-Tracy method and Fetkovich model.

An additional comparison was developed and published in Business Computing and Global Informatization (BCGIN), 2011 International Conference on, where was presented and analyzed a comparison of the three (van Everdingen-Hurst, Carter-Tracy and Fetkovich) traditional calculation models of water influx **(Heng-ru, Z., Ying, L., Shanghai 2011).** Then a detailed analysis of Fetkovich method, water influx calculation software was developed based on Fetkovich method. Finally, an example was developed using the software established.

Moreover, different projects have been performed in the last years searching for an alternative way to study and measure water influx. An example of this was a simple approach created to develop a new aquifer influx model for a finite aquifer system that admits a pseudo-steady-state flow regime **(Omeke, J. E, Nwachukwu A, Awo R.O, Boniface O & Uche, August 2011).** This work used an appropriate boundary condition and a solution was gotten, by which was possible a direct calculation of cumulative water influx, at a given time, without the use of superposition or pressure approximations or iterative means. In other to derive a solution under varying reservoir-aquifer boundary pressure the dependency of the boundary pressure with time was described as an exponential trend. Model validation included the comparison of the results with those obtained from Carter & Tracy, Fetkovich and Van Everdingen models. It is important to pointed out that Van Everdingen and Hurst Model were considered as the main models for the comparison while two cases examples were considered. Results from this study showed the agreement between the aquifer model proposed and the existing models during their execution, highlighting the apparently better performance of the proposed model in comparison with the results obtained from Carter & Tracy.

Finally, some studies about modeling fluid flow have been carried out with the aid and application of several software's in order to try to simulate the behavior of the response of an aquifer. Moreover, a paper **(Kehinde D., Ikwan U., Oghene N., Afolayan O., Awa C & Sell Petroleum Development Company. Nigeria, 2.013)** was developed with the aim to investigate the effect of gas water contact (GWC) boundary on the pressure transient behavior, concluding that the main effect is extremely related with the external aquifer radius. Accordingly, for an external aquifer radius equal to 3 and below, the pressure derivative response is similar to a constant pressure boundary response, while for an aquifer radius equal to 8 or higher the pressure response of the aquifer model can be represented with an infinite acting aquifer. The last investigation allowed to observe the importance of the aquifer parameters, especially the aquifer radius, and thus its sizes.

3. STATE OF THE ART

3.1 Water Drive Reservoir

Some of the most prolific oil fields in the world are water drive reservoirs. Feasibly the most distinguished example could be the East Texas field, where the final oil recovery in the East Texas field has been projected to be approximately 79% **(Roadifer, R. E. 24 February 1986).** As this example can show, water influx can be considered as one of the best sources of energy to improve an oil field recovery.

Petroleum field tends normally to be surrounded by water, and this fluid many time has the tendency to maintain the pressure during the production of the field. This special characteristic is also well known as water drive mechanism. During production, the differential of pressure generated tends to expand the waters contained into the aquifer, while its movement through the reservoir zone takes place. The incoming water helps drive the hydrocarbon to the producing wells, leading to improve the hydrocarbon recovery.

The level in which water drive can improve the final hydrocarbon recovery is strictly related with the aquifer size, degree of communication between the aquifer and the hydrocarbon zone, and finally the aquifer strength, that refers to the capacity of the aquifer to mitigate the reservoir's normal pressure decline.

3.2 Classification of Reservoir-Aquifer Systems.

Nowadays, hydrocarbon field around the world are normally produced under water drive mechanisms. Often this is called "natural water drive" in order to differentiate it from a secondary production mechanism, as for instance, artificial water drive that pretends to use water injections processes into the formation to improve hydrocarbon recoveries. The pressure disturbance generated by the production of the field, produce the pressure drop responsible for the movement of the fluid into the field, and thus the water into the aquifer. This response that create an entrance of some quantities of water into the reservoir, later appear into the production wells, and thus production history. The water coming from the aquifer is identified as water influx or water encroachment, which is accredited to the following situations:

- Expansion of the water in the aquifer.
- Compressibility of the aquifer rock.
- Artesian a flow where the water-bearing formation out crop is located structurally higher than the pay zone.

Reservoir-aquifer systems are usually classified on the following subsections:

3.2.1 Degree of pressure maintenance:

Depending of the degree of reservoir pressure maintenance generated by the aquifer, a natural water drive process can be qualitatively described as:

- The active water drive:

This kind of water drive mechanism is particularly characterized by the presence of a water influx rate that can equal to the reservoir total production rate. This kind of mechanisms are typically categorized by a regular and slow reservoir pressure decline. Whether a long production period is being carried out and reservoir pressure tends to remain constant, the reservoir voidage rate must be the same to the water influx rate.

- The partial water drive:

Mechanisms referred to one in which the water encroachment rate is appreciably less than the reservoir's fluid withdrawal rate. These reservoirs are called partial water drives.

- The limited water drive.

3.2.2 Outer Boundary Conditions:

Geological hydrocarbon formations are normally finite (a close system) but they can be classified also as infinite whether the alterations in the pressure at the hydrocarbon-water contact are not "felt" at the aquifer boundary. In fact, is this condition of finite or infinite aquifer what is going to define the behavior of the aquifer, while it can be subdivided as follows:

An infinite system refers to a system in which the pressure fluctuations at the hydrocarbon/aquifer boundary takes so much time to be "felt" at the outer boundary. The outer boundary is intentionally proposed with a constant pressure equal to initial reservoir pressure.

Moreover, in a finite system the aquifer outer boundaries are affected by the water influx generated due to the pressure drop into the field. Most important, fluctuations in the pressure at the outer boundaries start to be found over time.

3.2.3 Flow Regimes

During the production process of a field, the flow of fluid in the reservoir tends to move in different ways at different times, which is in several cases a strictly related to the shape and size of the reservoir. Flow regimes specify the basic reservoir features and stipulate the influence that the rate of water influx has into the reservoir. Moreover, the basic flow regimes are considered in terms of time, and when specifically they are being carrying out. The principal flow regimes for different time categories widely used in the reservoir engineering are as follow:

- Steady State.
- Semi (pseudo) Steady State.
- Unsteady State.

3.2.4 Flow Geometries

Reservoir-aquifer systems can be classified based on flow geometry as follow:

3.2.4.1 Edge- water drive

The aquifer exclusively feeds one side or flank of the reservoir. These types of reservoir-aquifer systems can be found in the following scenarios:

3.2.4.1.1 Linear Models

Linear model assumes the reservoir and aquifer are place alongside rectangular parallelepipeds. For edge-water drives, the thicknesses of the reservoir and aquifer are identical; the widths of the reservoir and aquifer are also the same. It can be possible to find this model in the following situations:

- Linear Infinite Aquifer: aquifer boundaries are not “felt”.
- Linear Finite Aquifer: aquifer boundaries are felt at certain time.

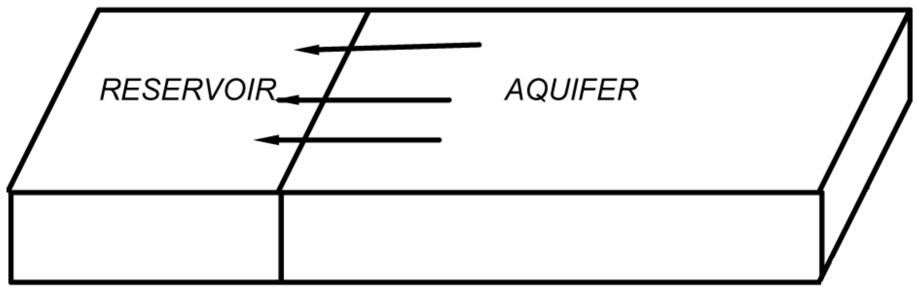

Fig. 3-1 Linear Aquifer model for an Edge-Water drive. [3]

3.2.4.1.2 Radial Models

- Van Everdingen and Hurst: The radial model assumes that the reservoir is a right cylinder and that the aquifer surrounds the reservoir. Flow between the aquifer and reservoir is strictly radial.

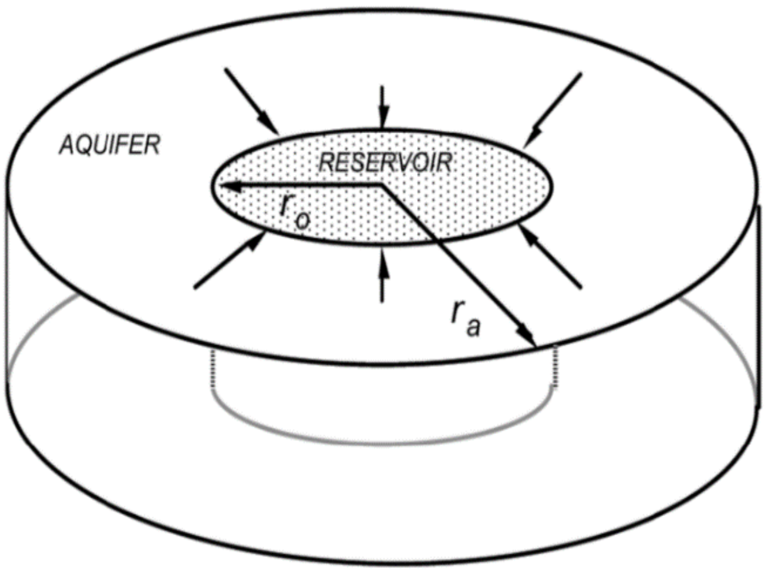

Fig. 3-2 Radial Aquifer Model. [3]

- Fetkovich Model: a model that proposed a pseudosteady-state aquifer productivity index and an aquifer material balance to represent the system compressibility. In this case, this statement can be applied for edge-water drive reservoir-aquifer systems.

3.2.4.2 Bottom-water drive

The aquifer under lays the reservoir and feeds it from beneath.

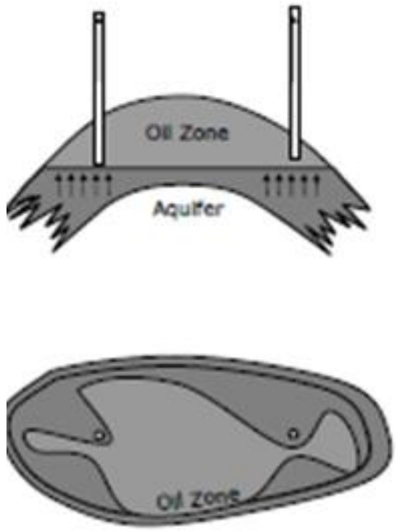

Fig. 3-3 Bottom- Water Drive. [3]

These types of reservoir-aquifer systems can be found in the following scenarios:

3.2.4.2.1 Coats Model

- Analytical Aquifer.
- Infinite Aquifer.
- Constant Terminal Rate at Interface.

3.2.4.2.2 Allard-Chen Model

- Numerical Aquifer Model.
- Finite-Infinite Aquifers.
- Constant Terminal Pressure at Interface.

3.2.4.3 Linear-water drive

Linear aquifers, either limited or essentially infinite, may be encountered in reservoir engineering practice. In areas where faulting fixes reservoir boundaries, the fault block reservoir may have an aquifer of limited extent whose geometry is best approximated as linear. An infinite linear aquifer can occur as a regional feature whenever water movement through the aquifer member is constrained to one direction. Such constraints can arise from major faults, facies changes or pinch-out of the member.

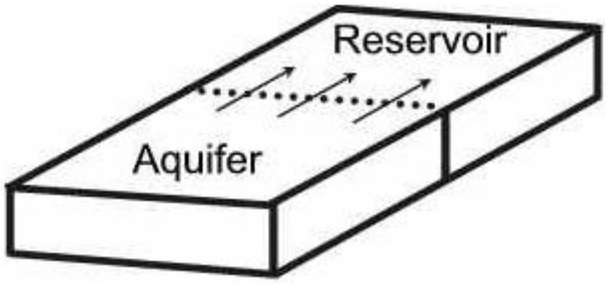

Fig. 3-4. Linear Aquifer Model. [4]

3.3 Water Influx Models

Several mathematical models have been developed through the history for predicting and understanding the behavior of the aquifer performance. This models are widely called Water Influx Models, referring to models that tries to simulate the complex behavior of the aquifer zone as the pressure changes at the reservoir-aquifer boundary.

Water influx models have been industrialized and intelligently integrated with simulation tools, to obtain a net and a more complete model that effectively can simulate the performance of the reservoir-aquifer system.

The main aquifer models available are:

- Van Everdingen-Hurst (VEH) model·
- Carter-Tracy model.
- Fetkovich model.
- Numerical Aquifer
- Schilthuis model.
- Small- or pot-aquifer model.

The most recognized model and more ones that are realistic are represented for Van Everdingen-Hurst, Carter-Tracy and Fetkovich models, since they attempt to represent, with a respectable accuracy, complexity of pressure changes within the aquifer zone and at the reservoir-aquifer boundary. As the production process of a field proceeds, the pressure differences between the reservoir and aquifer raises speedily and then the pressure differences tends to stabilize as the aquifer and reservoir eventually equilibrate. It is important to realize that the presence of this pressure difference who generates a water influx rate starting from zero value, grows steadily, reaching a maximum value and then dissipating it over time.

Respect to the flow regimes, those models that are based on studies done during unsteady-state flow regime are far more successful at capturing the real dynamics than other models. On the other hand, Schilthuis' steady-state model supposed aquifer pressure as a constant, while Van Everdingen-Hurst, which is considered as the most sophisticated of all these models, exposed a controversial realism.Furthermore, Van Everdingen-Hurst proposed a very complex model where charts and tables were needed to be consulted repeatedly to perform simple calculations. To contrast these difficulties in the execution of the Van Everdingen-Hurst model, Carter-Tracy and Fetkovich constructed models avoiding the limitations presented for Van Everdingen-Hurst model, specifically in the implementation of alternatives free of charts and tables, and escaping from the need to implement the superposition principle, implemented by Van Everdingen-Hurst model. These models, however, were only approximations to the more realistic one and based on simplifications of the Van Everdingen-Hurst model.

It is important to point out that numerical aquifers are represented by a one-dimensional row of cells within the simulation grid. The other types of aquifer, classed as 'analytic aquifers', are represented by computed source terms in the reservoir grid cells with which they connect.

3.3.1 Van Everdingen-Hurst (VEH) Model

Van Everdingen & Hurst (1949) presented a model that deals with radial and liner aquifers, proposing the application of Laplace transformation, as a solution for the diffusivity equation of the reservoir-aquifer system, considering a constant pressure at the boundary as boundary condition. Van Everdingen & Hurst (1949) expressed the flow supplied by the aquifer at the aquifer-reservoir contact using Darcy equation, as follow:

$$\mathrm{q} = \frac{2\pi f k h}{\mu}\left(r\frac{\partial p}{\partial r}\right)_{r_o} \tag{1}$$

Where:

- f : Factor describing a radial sector.
- k : Permeability.
- h : Thickness.
- μ : viscosity.
- r : radius.

The previous equation was re-written using the dimensionless variables definitions, as follow:

$$\left(r_D\frac{\partial p_D}{\partial r_D}\right)_{r_D=1} = \frac{q}{2\pi f k h \Delta p_0} \equiv q_D(t_D) \tag{2}$$

Where:

- $q_D(t_D)$: dimensionless flow supplied by the aquifer.
- Δp_0 : pressure drop at the boundary.
- t_D : dimensionless time.

The dimensionless flow supplied by the aquifer was supposed to be calculated at the aquifer-reservoir boundary. Moreover, the accumulated influx was expected to be found as the integral of the flow over time, as follow:

$$W_e \equiv \int_0^t q dt = \frac{2\pi f k h \Delta p_0}{\mu}\int_0^{t_D} q_D \frac{dt}{dt_D} dt_D \tag{3}$$

Expressing $W_D(t_D)$ as the integral of q_D regarding to t_D, the above equation can be simplified and rewritten, as follow:

$$W_e = U\Delta p_0 W_D(t_D) \tag{4}$$

Where:

- U : The influx constant of water into the aquifer.
- W_D: Accumulated dimensionless influx for a constant pressure drop at the boundary.

Van Everdingen & Hurst (1949) proposed a model theoretically correct, and was described as an exact solution of the hydraulic diffusivity equation (Matheus & Russel, 1967). Following the traditional flow regimes conditions applied for both radial and linear aquifers, Van Everdingen & Hurst (1949) model suggested to consider infinite aquifer during transient regime, aquifers sealed in the outer boundary as pseudosteady state regime, and aquifers with support of pressure in the outer boundary for a steady state regime. Most solutions calculation of $W_D(t_D)$ are represented by the Laplace space, where inversion can be done by numerical methods.

The main restrictions of Van Everdingen & Hurst (1949) model lies to the fact of using the principle of superposition (the Duhamel principle), as boundary pressure does not remain constant during the production of the reservoir. Accordingly, the application of superposition- the Duhamel principle can be showed as follow:

$$W_e = U\int_0^{t_D} q_D(t_D - \tau_D)\Delta p(\tau_D)d\tau_D \tag{5}$$

Or

$$W_e = U\int_0^{t_D} W'_D(t_D - \tau_D)\Delta p(\tau_D)d\tau_D \tag{6}$$

The last equation expressed the application of the Duhamel equation in the Van Everdingen & Hurst (1949) model, supposing to know the boundary pressure drop over time as $\Delta p(t) = p_i - p(t)$, and also the knowledge of the derivative of the classical solution of the model under study W'_D. Van Everdingen & Hurst (1949) proposed a discretization of the inner boundary condition, as for example, the boundary pressure, $p(t)$. Hence, the pressure curve could be divided into constant pressure intervals, as figure below can show:

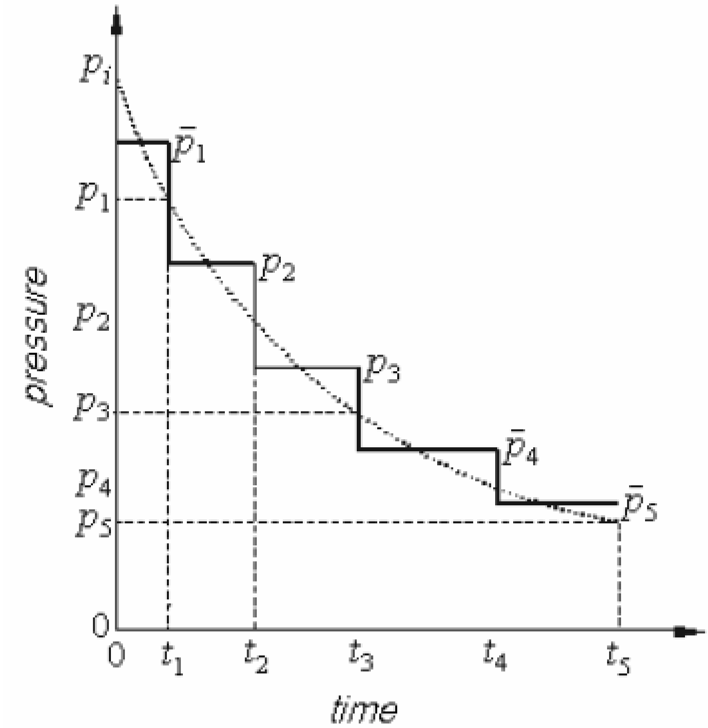

Fig. 3-1. Discretization of Boundary Pressure.[8]

By means of the pressure discretization, the Duhamel equation was re-written as follow:

$$W_e(t_{Dn}) = U \sum_{j=0}^{n-1} (p_i - \bar{p}_{j+1}) \left[W_D(t_{Dn} - t_{Dj}) - W_D(t_{Dn} - t_{Dj+1}) \right] \quad (7)$$

Where:

- $\bar{p}_{j+1}$: The mean pressure in each interval.

Replacing the mean pressure in the previous equation is possible to obtain the following expression of the accumulated influx:

$$W_e(t_{Dn}) = U \sum_{j=0}^{n-1} \Delta p_j \left[W_D(t_{Dn} - t_{Dj}) \right] \quad (8)$$

Where:

- Δp_j : Difference of mean pressure as $\bar{p}_j - \bar{p}_{j+1}$.

Following the discretization suggested above as a solution to the Duhamel principle, Van Everdingen & Hurst (1949) could shorten the use in computer programs. However, Van Everdingen & Hurst (1949) model required to calculate the term t_{Dn} for each programming step, which involved recalculations in all steps when the value of t_{Dn} was changed. Accordingly, calculations of accumulated influx at each new step of dimensionless time had to be done again, which resulted in a complex and tedious model.

3.3.2 Carter-Tracy Aquifer

Carter-Tracy aquifer model (1960) was developed to reduce the complexity of the most rigorous and original aquifer model established by Van Everdingen & Hurst (1949), which does not need the execution of superposition principle to calculate water influx. Carter-Tracy aquifer model is a very complete model, since it covers any flow geometry, as long as, the solution for the dimensionless pressure, as a function of time is known for the geometry of the aquifer being analyzed. This adjustment to several types of aquifer cases and the fact that is easy to program in a numerical level, becomes Carter-Tracy aquifer model in one of the most popular and used model.

In the Carter-Tracy model the value of the accumulated influx W_e is approximated by the following equation:

$$W_e(t_{D\,j}) = W_e(t_{D\,j-1}) + a_{j-1}(t_{D\,j} - t_{D\,j-1}) \tag{9}$$

Where

- a_{j-1} : A constant.
- t_D : dimensionless time defined for each aquifer geometry

In the interval between t_{Dj-1} and t_D, the previous equation assumes that the influx varies linearly with time.

The accumulated water influx can be expressed by the convolution integral, as follow:

$$W_e(t_{D\,j}) = U \int_0^{t_{D\,j}} \Delta p\ (t_D) dW_D(t_D - \tau) d\tau \qquad (10)$$

Where:

- U : The influx constant.
- $\Delta p\ (t_D) = p_i - p(t_D)$: The pressure drop at the boundary.
- $W_D(t_D)$: Dimensionless accumulated water influx
- τ: A dummy integration variable.
- *j :* Refers to time discretization.

Solving by the Laplace transformation, the expression for the constant a_{j-1} is found, as follow:

$$a_{j-1} = \frac{U\, \Delta p(t_{D\,j}) - W_e(t_{D\,j-1})\, p'_D(t_{D\,j})}{p_D(t_{D\,j}) - t_{D\,j-1}\, p'_D(t_{D\,j})} \qquad (11)$$

Replacing constant a_{j-1} results in:

$$W_e(t_{D\,j}) = W_e(t_{D\,j-1}) + \frac{U\, \Delta p\ (t_{D\,j}) - W_e(t_{D\,j-1}) p'_D(t_{D\,j})}{p_D(t_{D\,j}) - t_{D\,j-1} p'_D(t_{D\,j})}\ (t_{D\,j} - t_{D\,j-1}) \qquad (12)$$

For representing the dimensionless pressure at the boundary of an aquifer producing under constant flow Carter-Tracy method used the $p_D(t_D)$. The limit for the applicability of the Carter-Tracy aquifer model lies to the knowledge about the $p_D(t_D)$ function for the geometry of the given aquifer.

For Carter-Tracy aquifer model the value of the accumulated influx, W_e, should be assessed at the time and for the kind of flow regime present in that specific time. Hence, for each flow regime it is possible to find different values of accumulated influx, W_e.

Between the two main parameters that govern the behavior of the aquifer are the time constant (with the dimension of time):

$$T_c = \frac{U_w \cdot \phi \cdot C_t \cdot r_o^2}{K_a \cdot C_1} \quad (13)$$

Where:

- K_a: Aquifer permeability.
- ϕ : Aquifer porosity.
- C_t: Total (rock + water) compressibility.
- U_w: Viscosity of water in the aquifer.
- r_o: Outer radius of the reservoir (or inner radius of the aquifer).
- C_1: 0.008527 (metric, PVT-M); 0.006328 (field); 3.6 (LAB).

The aquifer influx constant (with the dimension of total influx per unit pressure drop), was expressed as follow:

$$\beta = c_2\, h\, \theta\, \phi\, C_t\, ro^2 \quad (14)$$

Where:

- h : The aquifer thickness.
- θ : The angle subtended by the aquifer boundary from the center of the reservoir, in degrees divided by 360.
- C_2: 6.283 (METRIC, PVT-M); 1.1191 (FIELD); 6.283 (LAB).

The time constant “T_c” is used to convert time “t” into dimensionless time “t_D” through:

$$t_D = \frac{t}{T_C} \quad (15)$$

Carter-Tracy model expressed the pressure drop at the aquifer boundary in terms of the dimensionless pressure influence function PI_D by:

$$Pa0 - \bar{P} = \frac{Q_a}{\beta} PI_D(t_D) \qquad (16)$$

Where:

- Q_a : The aquifer inflow rate.
- Pa0: The initial pressure of water in the aquifer.
- $\bar{P}$: The average water pressure on the aquifer/reservoir boundary.

The average inflow rate from the aquifer to a grid block over a simulator time interval is calculated as:

$$Q_{ai} = \alpha_i\{a - b[p_i(t+\Delta t) - p_i(t)]\} \qquad (17)$$

Where:

$$a = \frac{1}{Tc}\left\{\frac{\beta\,\Delta p_{ai} - W_a(t) PI_D{}'(t+\Delta t)_D}{PI_D(t+\Delta t)_D - t_D PI_D{}'(t+\Delta t)_D}\right\} \qquad (18)$$

$$b = \left\{\frac{\beta}{T_c[PI_D(t+\Delta t)_D - t_D PI_D{}'(t+\Delta t)_D]}\right\} \qquad (19)$$

Where:

- Δp_{ai}: Pressure drop.
- $PI_D{}'$: Pressure derivative of PI_D with respect to t_D.
- α_i: Area fraction for each connection.

3.3.2.1 Applicability of Carter-Tracy model

Carter Tracy model applies to *finite-acting and infinite-acting* aquifers, while Fetkovich applies only to finite-acting aquifers. The model is applicable to both radial and linear aquifers. Furthermore, Carter-Tracy aquifer model can be implemented to edge-water drive reservoirs only, but can be adapted to bottom drive reservoir under specific conditions.

3.3.3 Fetkovich Aquifer

The Fetkovich aquifer model **Fetkovich (1971)**, was developed to understand the behavior of the finite aquifers for radial and linear geometries, but also extended to study the performance of the aquifer during pseudosteady-state regime flows. This model was called an “approximate” model due to several uncertainties in its mathematical conception. However, as Carter-Tracy aquifer model, it has the facility to be applied in terms of programming, as it does not need the application of the superposition principle required by Van Everdingen & Hurst model (1949), who needed all calculations at each time-step done several times, resulting in long processing times. Fetkovich assumed a pseudosteady-state regime while the movement of water from the aquifer through the reservoir zone is taking place.

As Carter-Tracy model, nowadays, Fetkovich model is a well-known aquifer model used in numerical simulation due to its ease of application for studying the aquifer performance.

Fetkovich aquifer model adopts the same productivity argument used during the production process of a field. Hence, during the production of a reservoir zone surrounded by water the same premise of productivity is adopted through the term of productivity index (PI). The productivity index could acceptably describe the movement of water, or the water influx from the aquifer through the reservoir zone, expressed in terms of water influx rate by the pressure drop generated from the pressure difference between the average aquifer pressure and the pressure at the water-hydrocarbon boundary. The Fetkovich model neglects the effects of any transient period, which can be a good reason to differ from the behavior of other models when cases, as for instance, abrupt changes at the aquifer-reservoir interface are expected to occur.

However, in the majority of the cases studied by Fetkovich, the aquifer model offers an excellent approximation respect to the most rigorous technique exposed by Van Everdingen & Hurst model (1949), since in many cases pressure changes at the waterfront are gradual.

Considering two simple equations, Fetkovich aquifer model has developed a rationally aquifer model based on the productivity index (PI) of the aquifer, being analogous to the productivity index used to describe the movement of oil or gas into a well.

$$Q_w = \frac{\mathrm{d}W_e}{\mathrm{d}t} = J\,(P_a - P_r) \qquad (20)$$

Where

- J : Aquifer productivity index.
- P_a : Aquifer average pressure.
- P_r : The pressure in the reservoir-aquifer boundary.

Applying the material balance equations into the previous formula, the following can be presented as:

$$P_a = p_i\left(1 - \frac{W_e}{W_{ei}}\right) \qquad (21)$$

Where:

- W_{ei} : Maximum influx *.*

The maximum influx represent the maximum volume of water that sealed aquifer can supply, corresponding to the expansion of water in the aquifer when the pressure decreases from p_i to zero, and can be expressed as follow:

$$W_{ei} = C_t W_i p_i \tag{22}$$

Due to the separation of the variables, the equation can be integrated from $t = 0$ (when $W_e = 0$ and $p_a = p_i$) as follow:

$$-\frac{J\,p_i}{W_{ei}}\int_0^t dt = \int_{p_i}^{p_a}\frac{dp_a}{p_a - p} \tag{23}$$

The solution proposed by Fetkovich results as follow:

$$p_a - p = (p_i - p)e^{-\frac{J\,p_i}{W_{ei}}t} \tag{24}$$

From this equation Fetkovich proposed a model based on some mathematical ambiguities, as it was allowed to maintain constant the pressure at the boundary, while is well known that pressure depends on time.

Replacing the last equation into the first one, the following results:

$$q = J(p_i - p)e^{-\frac{J\,p_i}{W_{ei}}t} \tag{25}$$

This equation schematizes the movement of the water flow through the reservoir as a function of time and the pressure drop at the boundary. This equation is general and does not depend on aquifer geometry. By integrating it, the following results:

$$W_e = \int_0^t q\,dt = J(p_i - p)\int_0^t e^{-\frac{J\,p_i}{W_{ei}}t}\,dt \tag{26}$$

Or

$$W_e = \frac{W_{ei}}{p_i}(p_i - p)\left[1 - e^{-\frac{J\,p_i}{W_{ei}}t}\right] \tag{27}$$

Even due to its mathematical inconsistencies, Fetkovich aquifer model quickly becomes in a popular method, since its ease of numerical programming. Following a discretization of the last equation, Fetkovich proposed an expression that could be used to represent the pressure variation at the boundary, resulting in the below expression, by which it is possible to obtain the value of the influx volume at any given time interval Δt_n.

$$\Delta W_{en} = \frac{W_{ei}}{p_i}\left(p_{a_{n-1}} - p_n\right)\left[1 - e^{-\frac{J\,p_i}{W_{ei}}\Delta t_n}\right] \tag{28}$$

Where:

$$p_{a_{n-1}} = p_i\left(1 - \frac{1}{W_{ei}}\sum_{j=1}^{n-1}\Delta W_{ej}\right) = p_i\left(1 - \frac{W_{e_{n-1}}}{W_{ei}}\right) \tag{29}$$

And

$$p_n = \frac{p_{n-1} + p_n}{2} \tag{30}$$

For the productivity index calculation is important to take into account the type of outer aquifer boundary that is present. Accordingly, the equations to be implemented for various reservoir/aquifer boundary conditions and aquifer geometries are as follow:

Aquifer Boundary	J for Radial Flow (STB/D-psi)	J for Linear Flow (STB/D-psi)
Finite, no flow	$J = \dfrac{0.00708kh(\frac{\emptyset}{360})}{\mu[\ln(r_a/r_r) - 0.75]}$	$J = \dfrac{3(0.001127)kwh}{\mu L}$
Finite, constant Pressure	$J = \dfrac{0.00708kh(\frac{\emptyset}{360})}{\mu \ln(r_a/r_r)}$	$J = \dfrac{0.001127kwh}{\mu L}$
Infinite	$J = \dfrac{0.00708kh(\frac{\emptyset}{360})}{\mu \ln \sqrt{\frac{0.0142kt}{\emptyset \mu C_t {r^2}_r}}}$	$J = \dfrac{kwh}{1{,}000\mu \sqrt{\frac{0.0633kt}{\emptyset \mu C_t}}}$

Tab. 3-1 Productivity Index Equations used for Fetkovich Model[10]

Where:

- $\emptyset$: Angle of aquifer incidence.
- k : Aquifer permeability.
- h : Aquifer thickness.
- μ : Aquifer viscosity.
- L : Linear aquifer length.
- r_a : Aquifer radius.
- r_r : Reservoir radius.
- w : Linear aquifer width.
- t : Time.

Accordingly with the above table, aquifer properties must be used to calculate aquifer productivity index.

3.3.3.1 Applicability of Fetkovich Model

Fetkovich model applies to *finite-acting* aquifers. The model is applicable to both radial and linear aquifers. Furthermore, Fetkovich aquifer model applies to edge-water and bottom-water drive reservoirs. In edge-water drive, water influx occurs around the flanks of the reservoir, while in bottom-water drive the reservoir is underlain by the aquifer which influxes vertically into the reservoir. Moreover, both methods provide simple and more direct methods for calculating cumulative water influx accurately.

Nomenclature

- P_r= Boundary pressure, psi.
- W_i= Initial volume of water in the aquifer, bbl.
- P_i= Initial pressure at the aquifer, psi.
- Ct= total compressibility, 1/psi.
- E_w= Aquifer influx rate, bbl/day.
- P_a= Average pressure in the aquifer, psi.

3.3.4 Numerical Aquifer Model

A numerical aquifer is modeled on the simulation tool used in this investigation by one-dimensional row of cells, trying to represent and understand the performance of the model aquifer studied. A one-dimensional set of cells were selected to characterize the aquifer, which must then be connected to that specific part of the cells representing the reservoir zone.

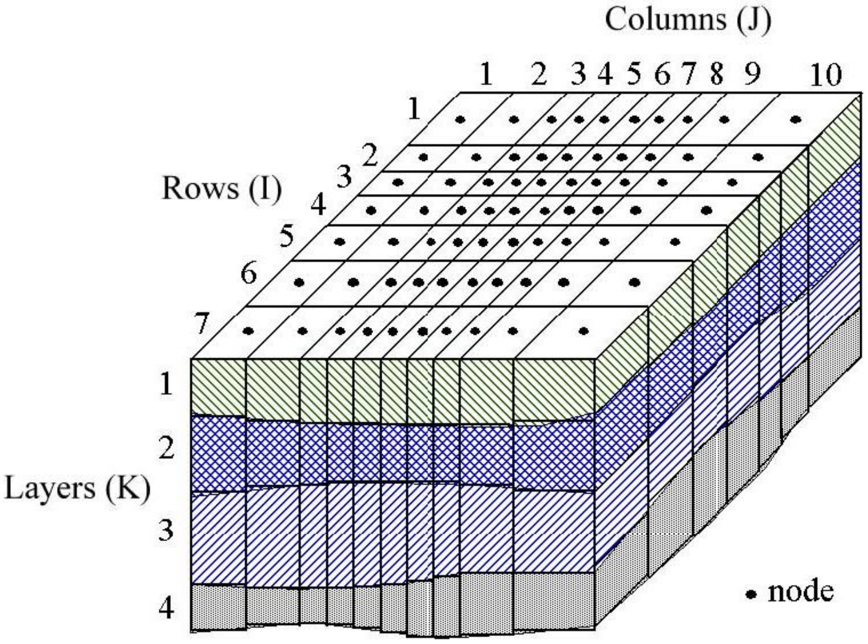

Fig. 3-5 Numerical Aquifer Model. [5]

To characterize appropriately an aquifer through a numerical aquifer model, all the principal parameters of the aquifer must be used. Between the parameters to take into account for the properly characterization of an aquifer, is important to mention the length, cross-sectional area, porosity, permeability, initial pressure, depth, PVT and saturation table numbers, which are used for the numerical model to build the appropriate aquifer grid blocks into the simulation tool. Using the correct keyword in the reservoir simulation tool, it can be possible the construction and representation of the aquifer. The properties that are not given to the simulation tool for building grid blocks are defaulted by the software using the information supplied into the "GRID" and "EDIT" sections. These sections provide independence of the aquifer properties from its actual position within the grid.

As already was mentioned, the numerical aquifer must be connected by using the appropriate keyword to a face of the reservoir, being the first cell of the aquifer linked by non-neighbor connections to the face of the reservoir. All aquifer cells are isolated from the grid except for those connected to the reservoir cells. For cases in which dual porosity is being considered, the aquifer should be placed in the lower half (fracture zone) of the grid. The one dimensional rows of cells must have a cross-section, length and depth, losing interest in any other information about their shape. The aquifer pore volume needs to be calculated from the parameters just mentioned and using the following equation:

$$\mathrm{PORV_i} = \mathrm{PORO_i} * \mathrm{LENGTH_i} * \mathrm{AREA_i} \tag{1}$$

Where:

- PORV_i : Pore volume of the aquifer cell.
- $\mathrm{PORO_i}$: Porosity of the aquifer cell.
- LENGTH_i: Length of the aquifer cell.
- AREA_i: Area of the aquifer cell.

Into the aquifer cell, porosity is calculated from the grid block section whether the value is not supplied to the software. Moreover, any other intention to impose a pore volume values by means of a “MULTPV” keyword in the “GRID” or “EDIT” sections will not apply to the numerical aquifer cells. Accordingly, any edits for the pore volume values in “EDIT” section will be ignored in the construction of a numerical aquifer, and the pore volume calculation is strictly carried out by the formula previously shown.

4. METHODOLOGY

Due to most hydrocarbon reservoirs around world are normally surrounded by water and the fact that their optimal strategy of development many time lies into the response of the aquifer zone and thus into the model used to represent the aquifer performance by means of a simulator tool, comes the need not only to study further the aquifer models developed through the story, but also to realize whether they can characterize an aquifer in the same manner, and when they start to differ between them.

As already discussed in the previous chapter the aquifer models available in the technical literature were developed under different hypothesis. The history matching phase of a water drive reservoir requires a proper calibration of the aquifer model parameters in order to obtain reliable results, and probably, the selection of the more suitable aquifer model. The aim of the study was a comparison between the behavior of the aquifer models in order to verify whether they can be considered equivalent (or at least interchangeable) or not, and when their behaviors diverge. Two different analytical aquifer models were considered in the current study: Carter-Tracy and Fetkovich, since consistently matched the objectives of this study, but also for their availability in the simulator tool. Given a proper calibration of the models, and in order to better focus the analysis on the aquifer parameter an extremely simplified reservoir model was considered. Furthermore, a number of different scenarios where generated and analyzed combining production constraints (constant bottom hole pressure and constant rate) and detailed sensitivity analysis on aquifer parameters.

Subsequently, an attempt of comparing the so called numerical aquifer to analytical was performed; results will be discussed in the following paragraphs.

The analyses were performed adopting the software ECLIPSE 100 (commercialized by Schlumberger).

4.1 Workflow

The methodology in order to obtain a success comparison between the aquifer models selected will be structured in the following way:

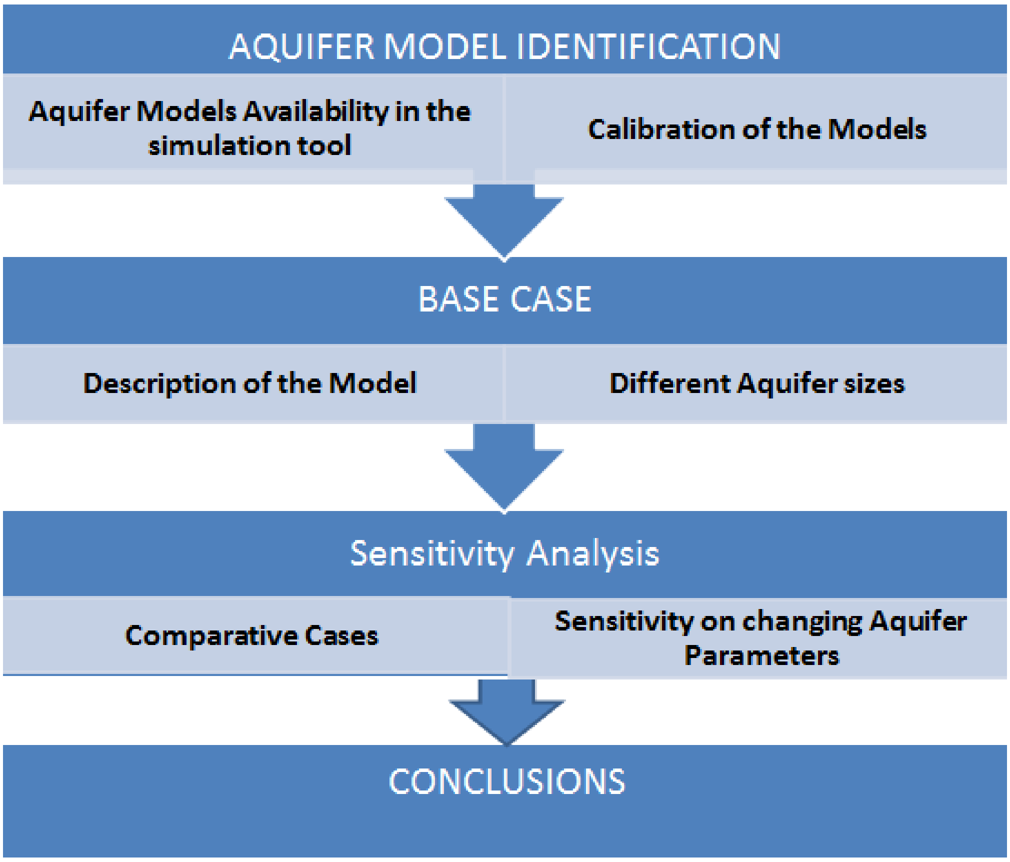

Fig. 4-1 Workflow.

4.2 Aquifer Model Identification

It was identified the aquifer models available in the simulation tool (ECLIPSE) and previously carried out a proper calibration of the models. The calibration was based in the principal parameters of each aquifer model. As it was discussed the analytical aquifer models selected were Fetkovich and Carter Tracy. For Carter Tracy aquifer model was necessary to establish coherent values in terms of pore volume of the aquifer, calculated from each different values of aquifer radius. Moreover, since Fetkovich used an inflow equation to model water influx from the aquifer to the reservoir, it was necessary to calculate the productivity index of the aquifer for each different values of aquifer radius.

4.3 Base Case

4.3.1 Description of the Model

The comparison study between the aquifer models selected to analyze will be originated from the development of a base case composed of a producing well in the middle of the reservoir-aquifer system; a reservoir zone, an ideal Bottom-Water drive mechanism, and the following features:

- Type of aquifer: Radial
- Dimensions: 1 cell 50x50x100 meters
- Pressure at the datum: 300 bars
- Porosity: 20%
- Compressibility: 3.98x10-5
- Thickness: 200m
- Reservoir Radius: 500m
- Datum Depth: 3000m

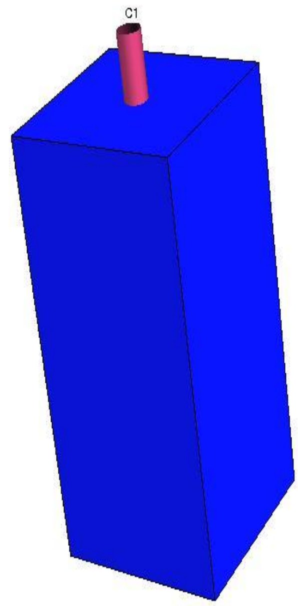

Fig. 4-2 Base Case Aquifer Model

The parameters shown above were used to perform the initial “calibration” of the models till confirm whether they were following a similar response.

4.3.2 Different Aquifer Sizes

The base case was performed for different bottom-aquifer sizes, in order to represent the extremely simple model selected, considering different aquifer size and then analyzing the possible effects in the implementation of the aquifer models. Accordingly, several reservoir sizes were taking into account:

rD	ra (m)
1.5	750
2	1000
2.5	1250
3	1500
3.5	1750
4	2000
5	2500
6	3000
8	4000
10	5000

Tab. 4-1. Aquifer sizes.[2]

The cumulative water influx and water influx rate have been plotted versus time for different aquifer radius in order to verify whether the models follow a similar trend. The picture below shows the behavior of Fetkovich and Carter-Tracy aquifer models for a drainage radius equal to: 2, 4, 6 and 10; that correspond to an aquifer radius of 1000, 2000, 3000 and 5000 meters, respectively.

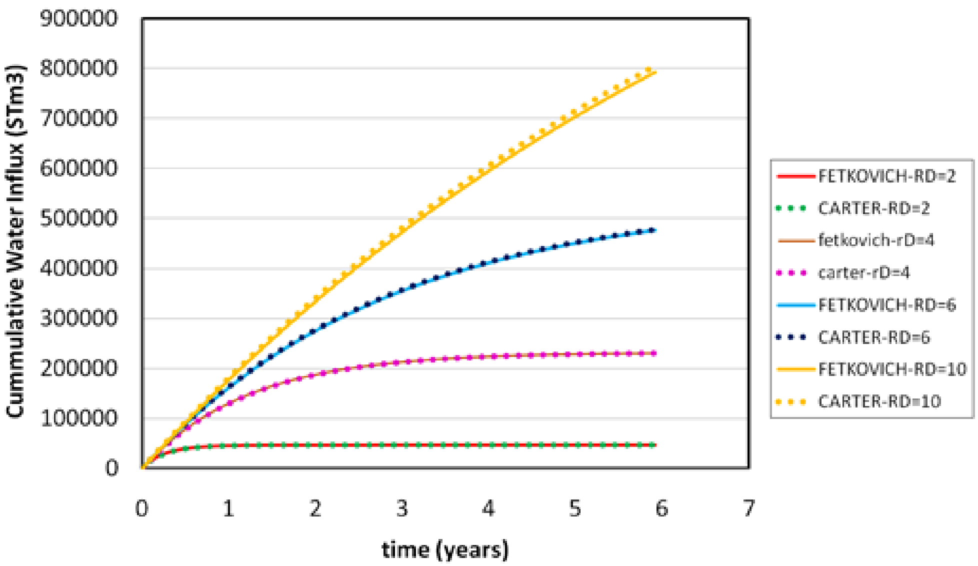

Fig. 4-3. Cumulative Water Influx for different aquifer sizes

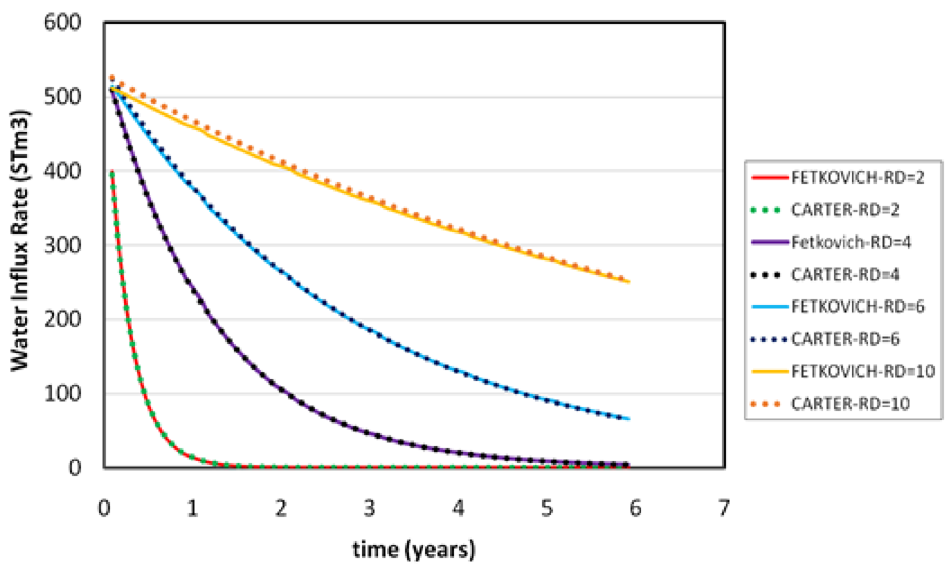

Fig. 4-4. Water Influx Rate for different aquifer sizes.

The same response was observed for both analytical models implemented when trying to represent the same aquifer system. As it could be appreciated in the graphs above, aquifer models were plotted in terms of cumulative water influx and water influx rate versus time. Both Fetkovich and Carter-Tracy showed a similar response in terms of aquifer performance (cumulative water influx and water influx rate vs. time), being verified also when different aquifer sizes were imposed.

5. SENSITIVITY ANALYSIS

5.1 Comparative cases

In this section have been developed several tests in order to verify the behavior of the aquifer models between them trying to recognize any circumstance under which they can expose a dissimilar performance. The main parameters to take into account for the comparative cases have been divided in two production constrains. Firstly a Constant bottom-hole pressure has been fixed in order to change aquifer parameters and observe the response of the models; secondly a constant flow rate has been established with the same purpose. A schematic embodiment of the comparative cases can be seen in the figure below.

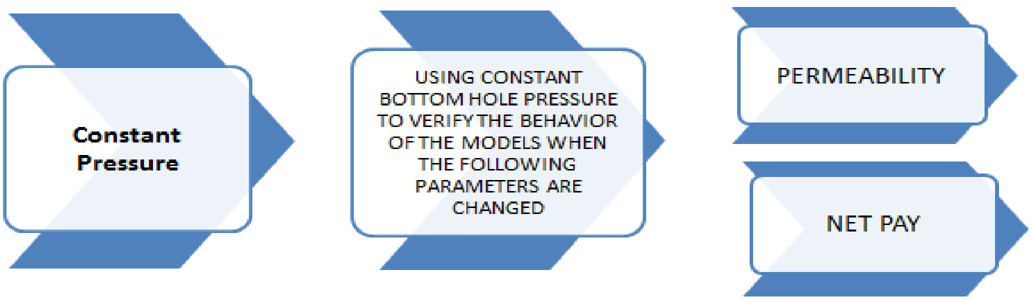

Fig. 5-1. Constant Pressure workflow

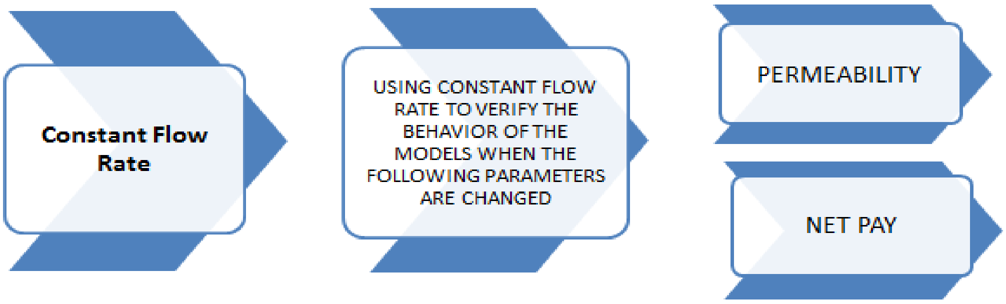

Fig. 5-2. Constant Flow Rate workflow.

During the analysis has been pointed out the aquifer performance (cumulative water influx and water influx rate) through time and the effects to change (increased or decreased) a specific parameter (permeability and net pay).

Moreover, because Fetkovich neglects the early transient time period in their calculations, it is expected that for small values of permeabilities the performance of this model diverges from Carter Tracy, decreasing the Fetkovich's outputs.

Nonetheless, assuming that the aquifer flow behavior obeys Darcy's Law, Fetkovich model uses an aquifer productivity index directly related with the thickness of the aquifer, while Carter-Tracy uses it to calculate the total volume of water present in the aquifer. Accordingly, bearing in mind that the thickness of the aquifer plays an important role in the calculation of the mentioned parameters, it could be expected observe changes into the response of the models.

5.1.1 Constant Bottom Hole Pressure

Considering a constant value for the bottom-hole pressure as the method of controlling the well into the reservoir, it could be possible to change some geometrical and geological parameters to observe the response of the model.

5.1.1.1 Changing Permeability values

Remembering that Fetkovitch model is quite affected by the effects of any transient time period, as it corresponds to a pseudo-steady state model, changes in permeability values could play a very important role during its implementation. The permeability can affect (increase or reduce) the duration in time of the flow regime during which the aquifer model is being applied. Hence, as permeability values decrease the effects of any transient period over the aquifer model response could be higher. Accordingly, a comparison scenario has been established between the base case (k=100mD) and additional situations with higher and smaller values of permeability. Following a logarithmic increment the ranges taken for changing permeability values is 1, 10, 100(Base case) and 1000mD.

For a permeability value of *k=1mD*:

a) Cumulative Water Influx:

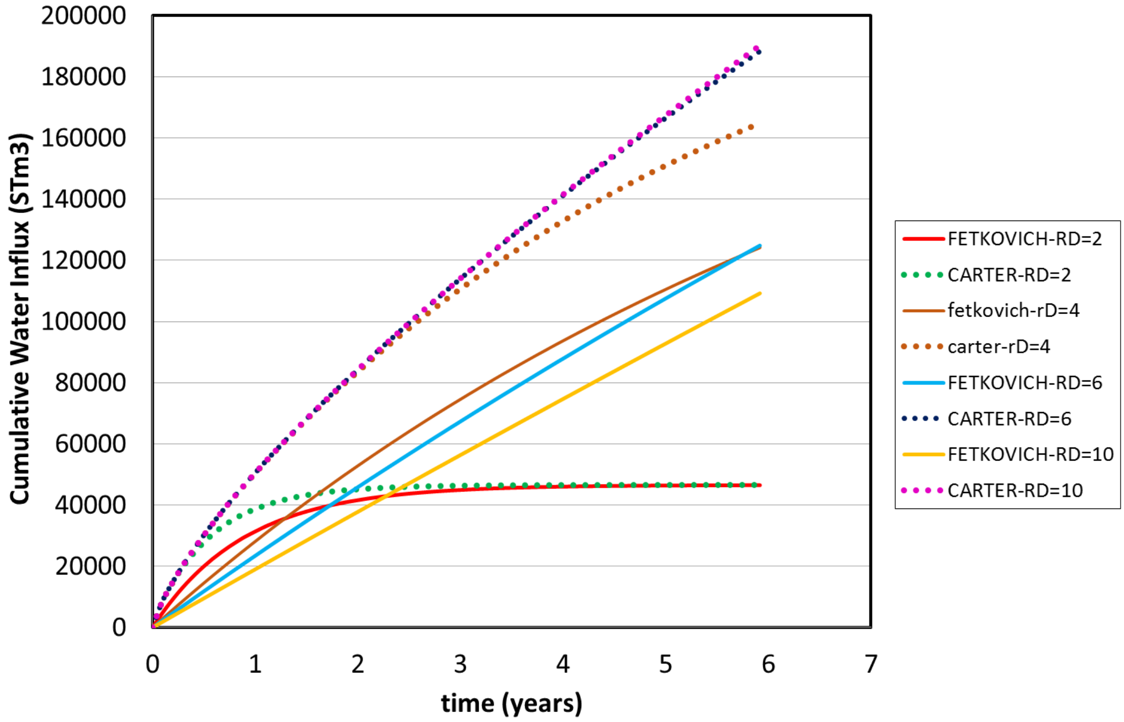

Fig. 5-3. Cumulative Water Influx for permeability value 1mD.

b) Water Influx Rate:

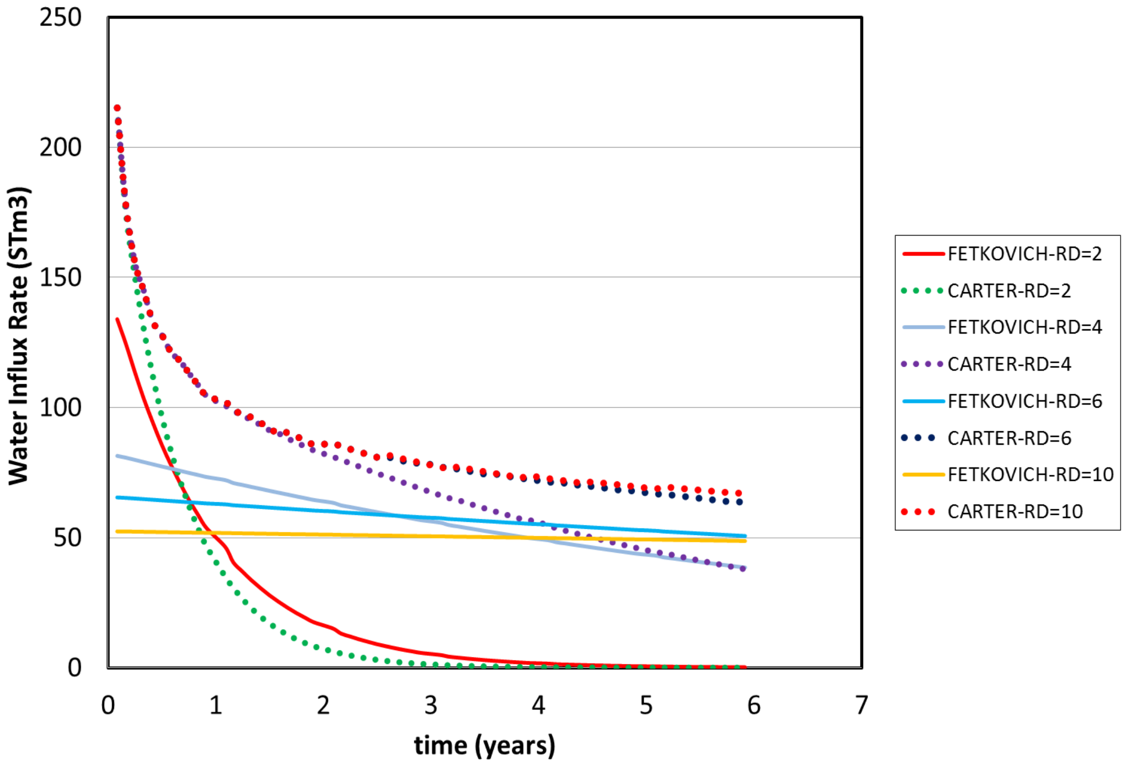

Fig. 5-4. Water Influx Rate for a permeability value 1mD.

For a permeability value of *k=10mD*:

a) Cumulative Water Influx

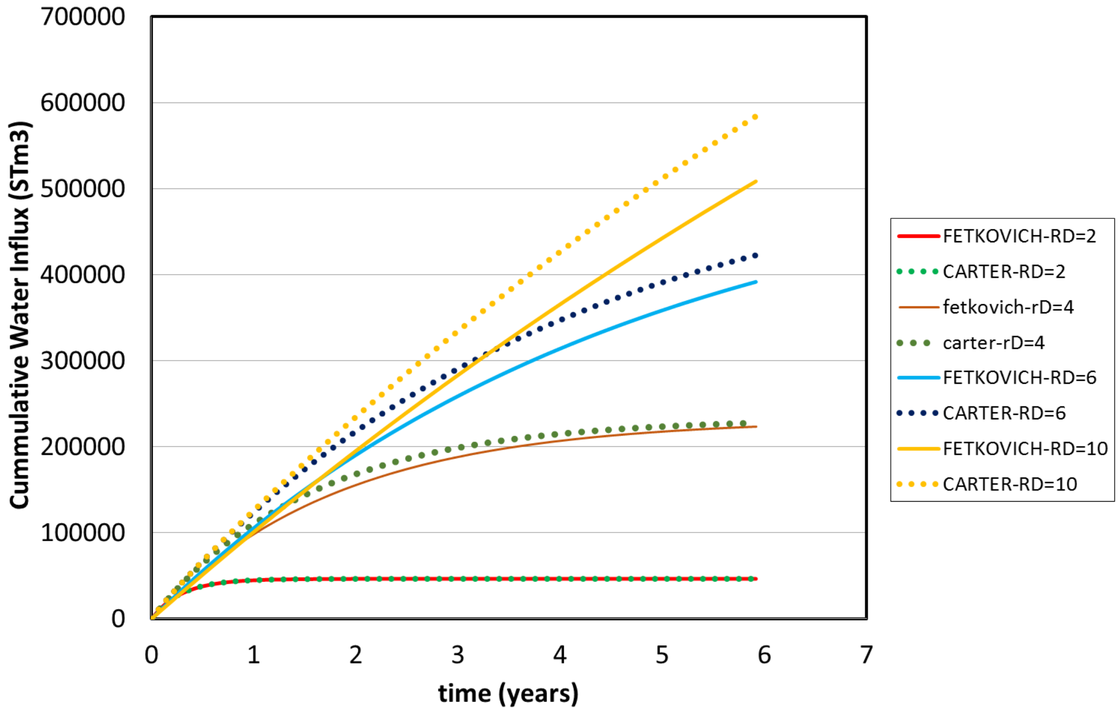

Fig. 5-5. Cumulative Water Influx for permeability value 10mD.

b) Water Influx Rate:

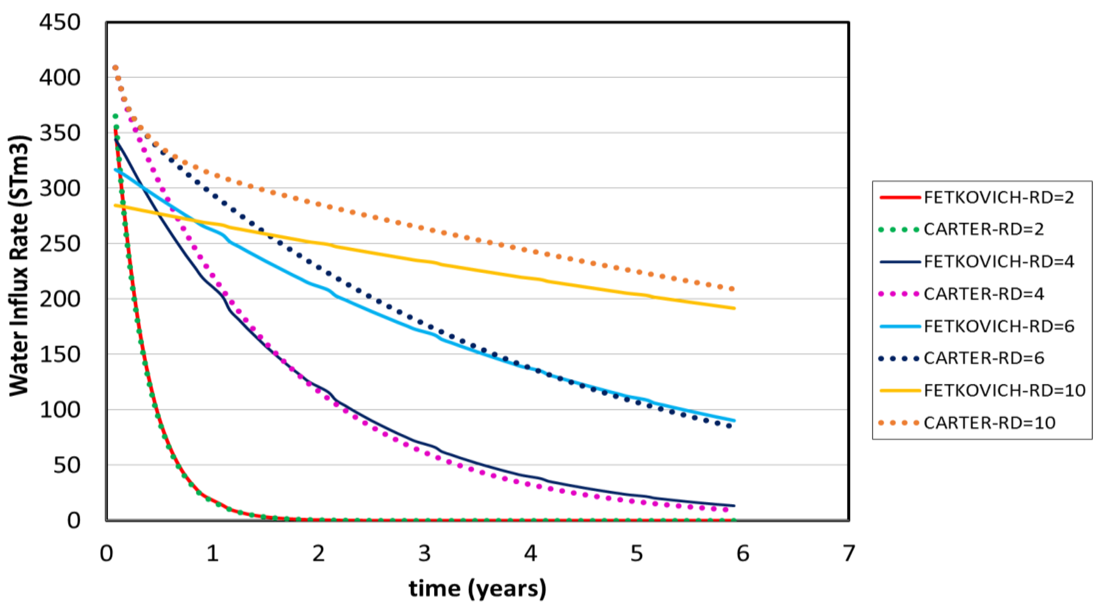

Fig. 5-6. Water Influx Rate for a permeability value 10mD.

For a permeability value of *k=100mD* (Base case):

a) Cumulative Water Influx

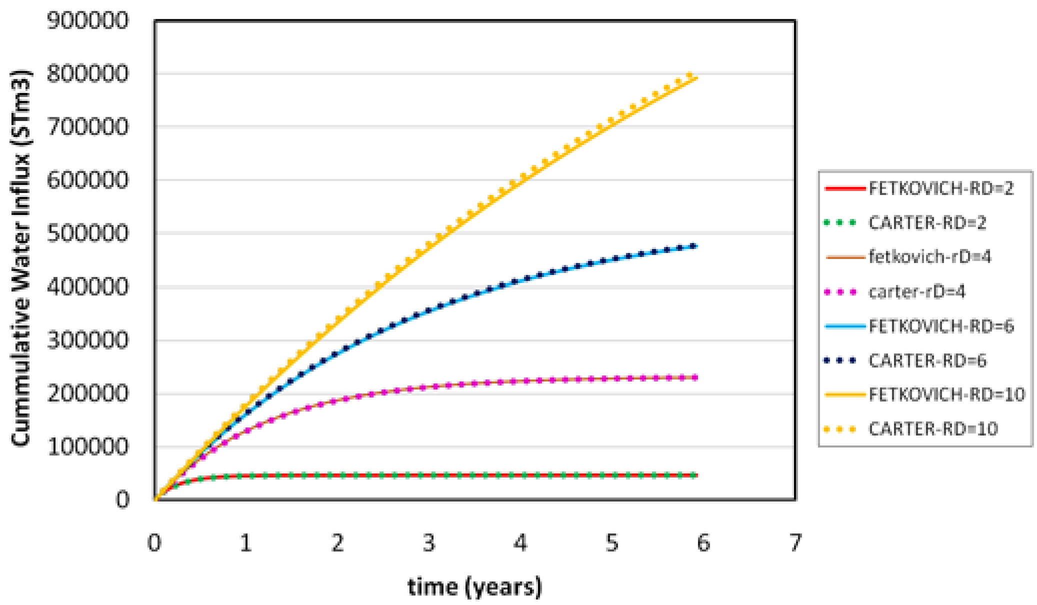

Fig. 5-7. Cumulative Water Influx for permeability value 100mD.

b) Water Influx Rate:

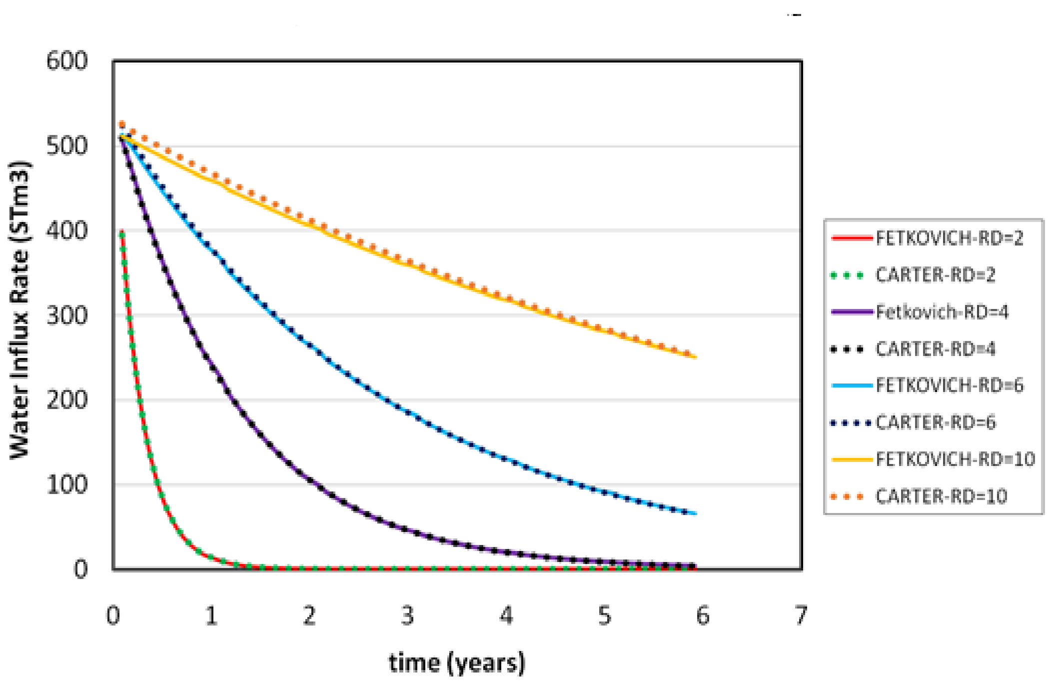

Fig. 5-8. Water Influx Rate for a permeability value 100mD.

For a permeability value of *k=1000mD*:

a) Cumulative Water Influx

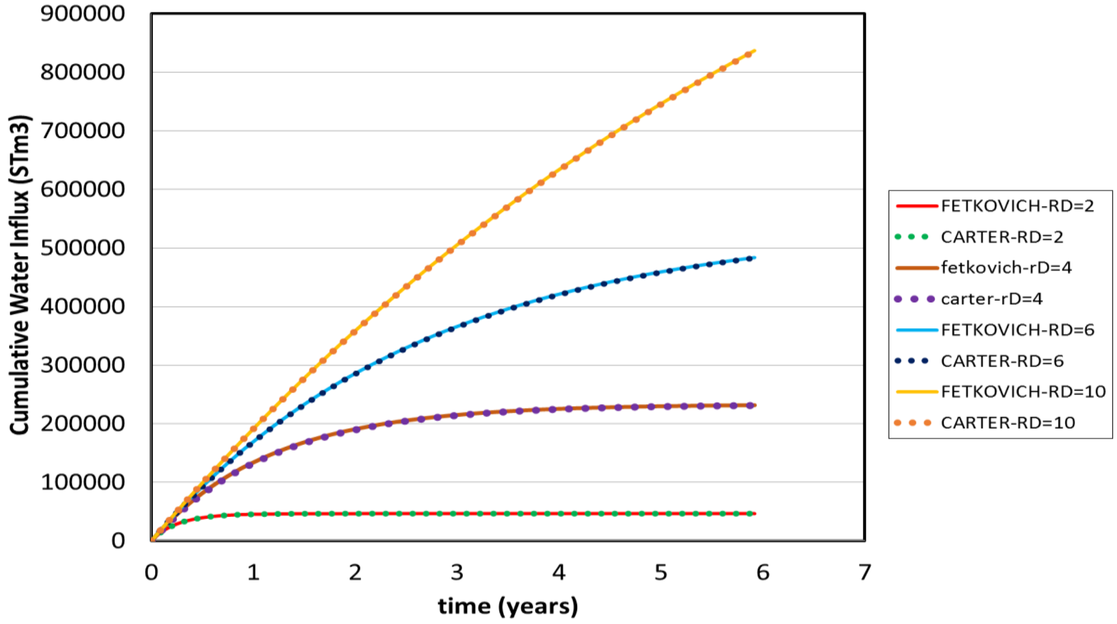

Fig. 5-9. Cumulative Water Influx for permeability value 1000mD.

b) Water Influx Rate:

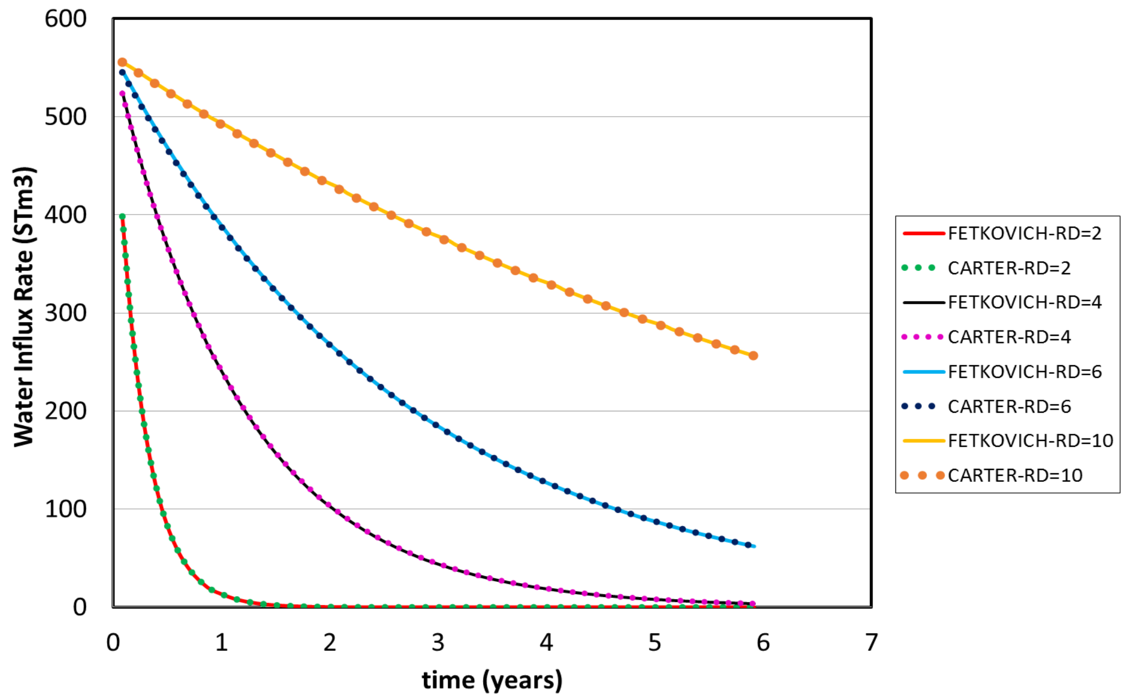

Fig. 5-10. Water Influx Rate for a permeability value 1000mD.

It should be pointed out that for small values of permeability in the system aquifer + reservoir, the behavior between Fetkovich and Carter-Tracy showed strong discrepancies between them for both aquifer parameters (cumulative water influx and water influx rate), due to the fact that Fetkovich does not consider the transient time that is decidedly present when the permeability becomes smaller into the system.

5.1.1.2 Changing Net Pay values

For a net pay value of 200 meters (Base case):

a) Cumulative Water Influx:

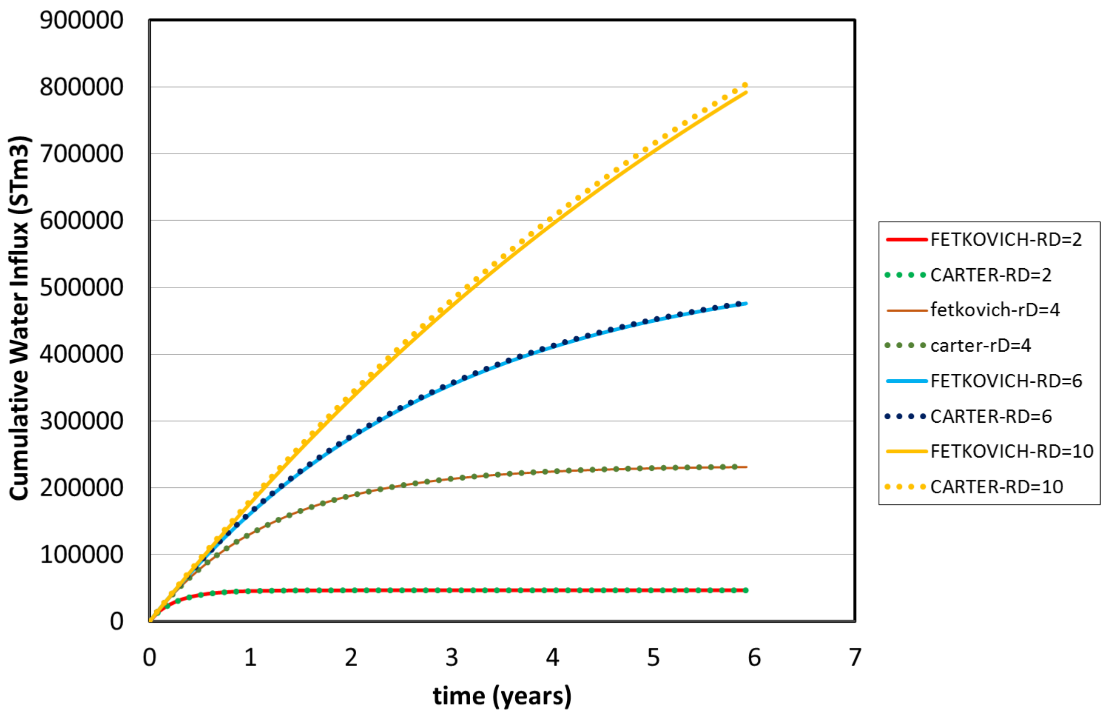

Fig. 5-11. Cumulative Water Influx for a net pay value of 200m.

b) Water Influx Rate:

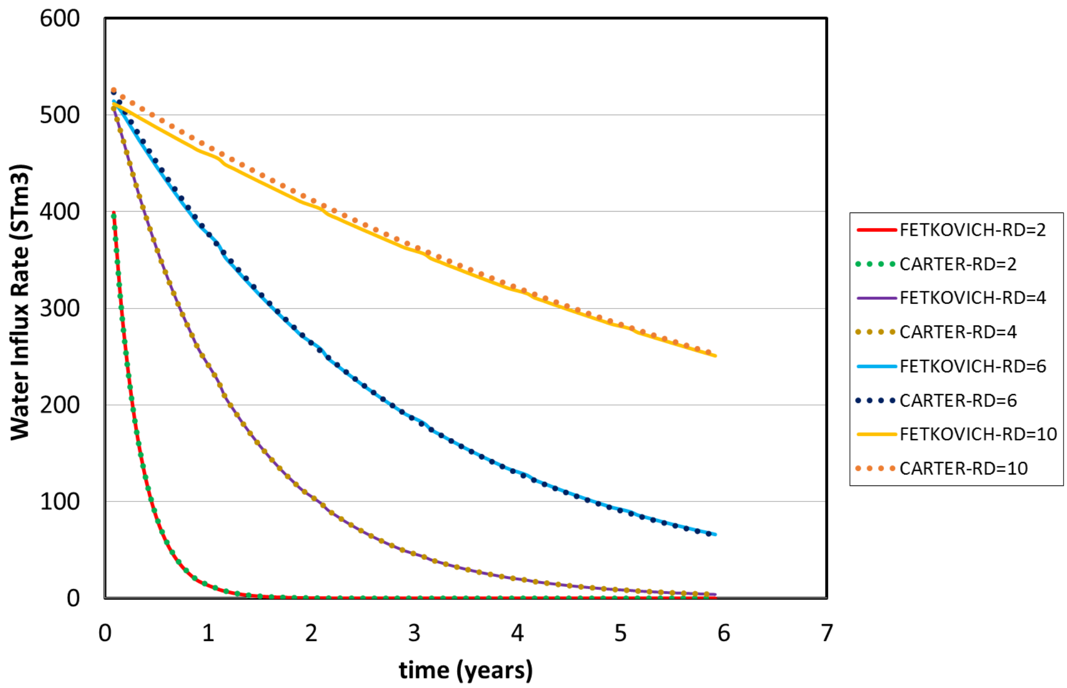

Fig. 5-12. Water Influx Rate for a net pay value of 200m.

For a net pay value of 500 meters (Base case):

a) Cumulative Water Influx

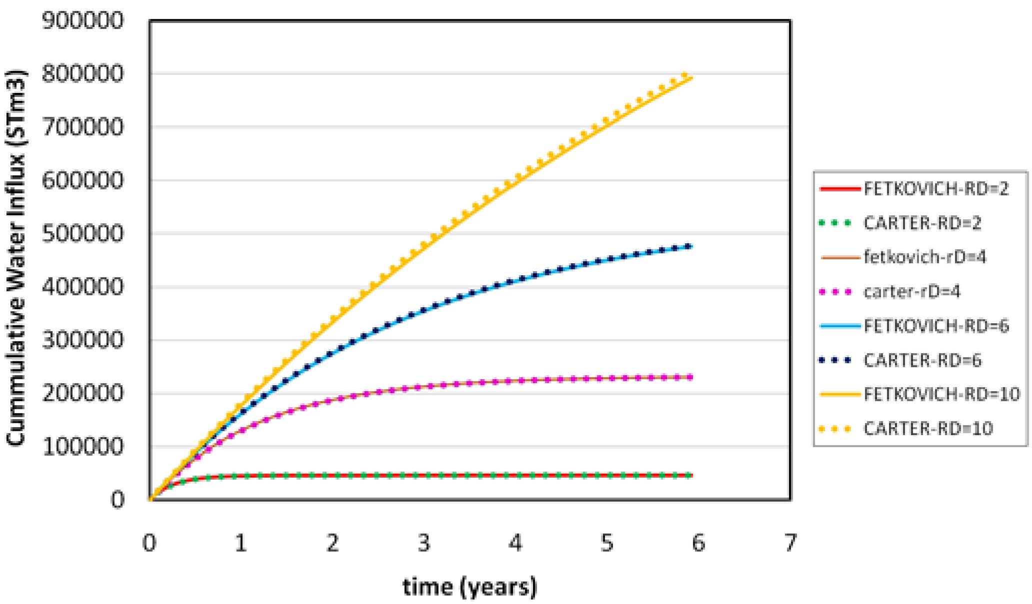

Fig. 5-13. Cumulative Water Influx for a net pay value of 500m.

b) Water Influx Rate:

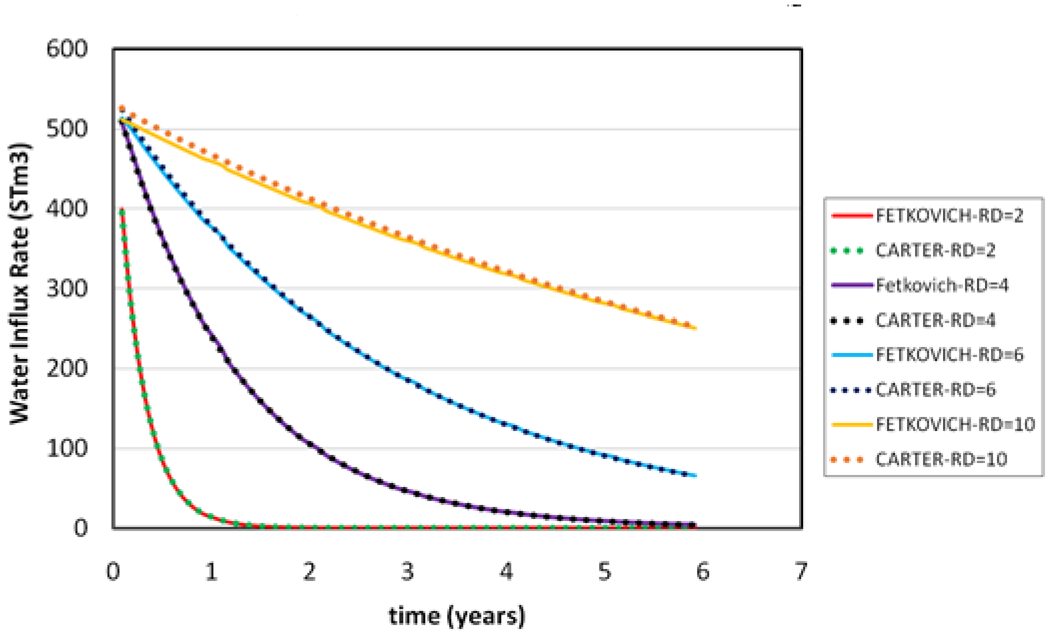

Fig. 5-14. Water influx Rate for a net pay value of 500m.

For a net pay value of 1000 meters (Base case):

a) Cumulative Water Influx:

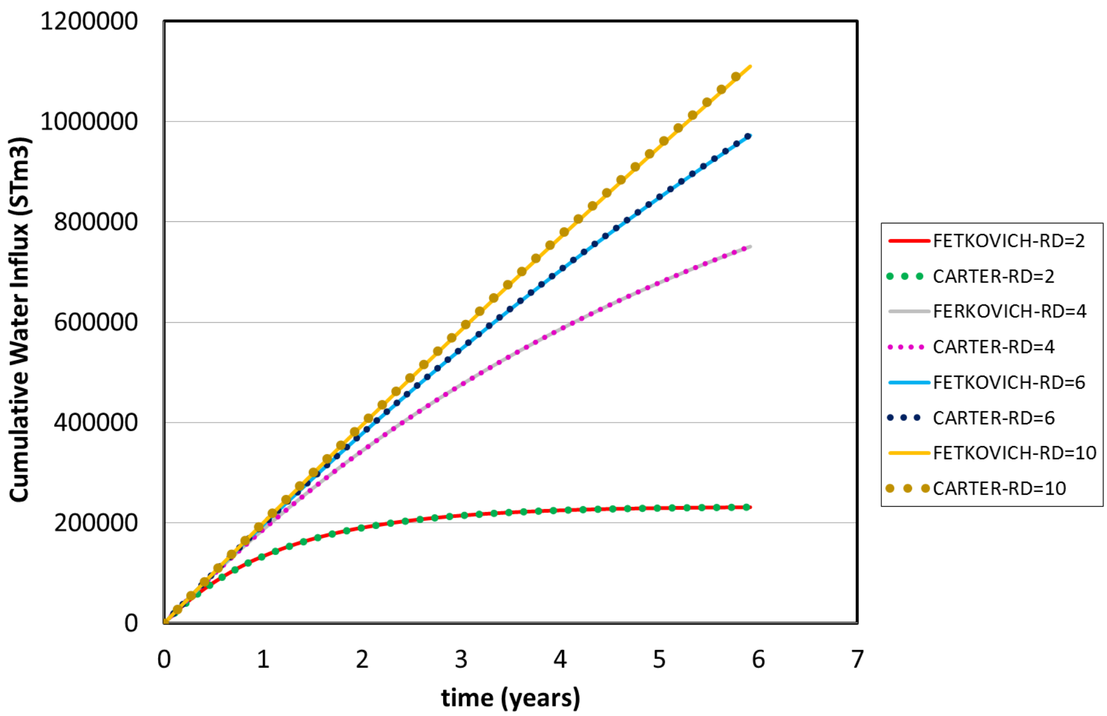

Fig. 5-15. Cumulative Water Influx for a net pay value of 1000m.

b) Water Influx Rate:

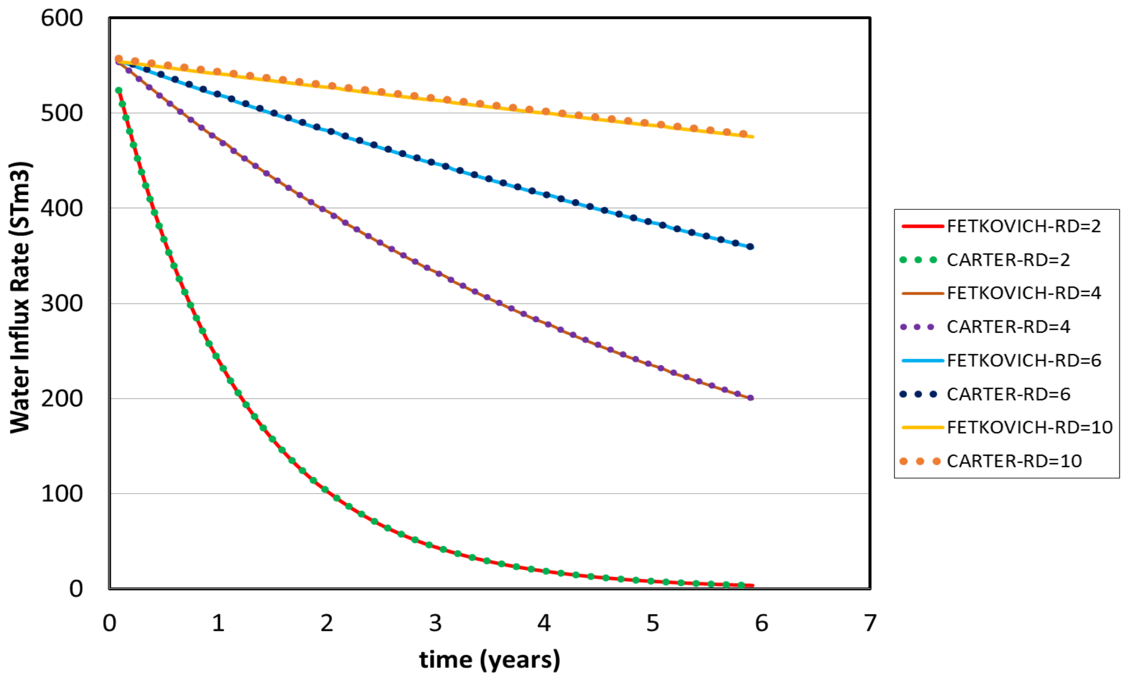

Fig. 5-16. Water Influx Rate for a net pay value of 1000m.

For a net pay value of 10000 meters:

a) Cumulative Water Influx:

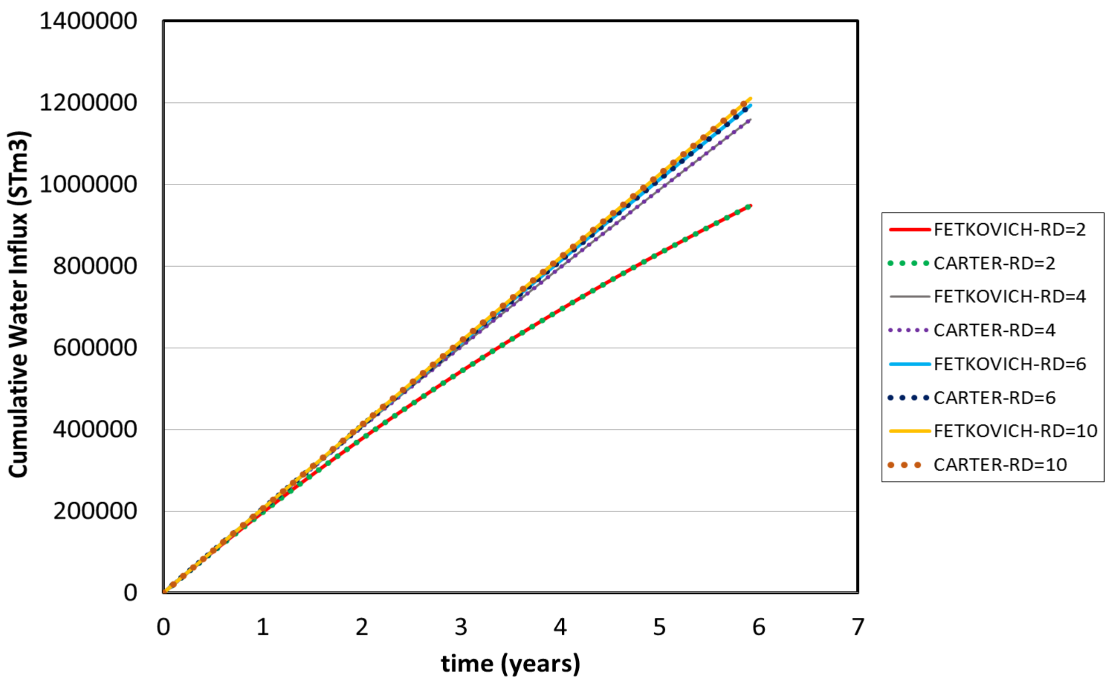

Fig. 5-17. Cumulative Water Influx for a net pay value of 10000m.

b) Water Influx Rate:

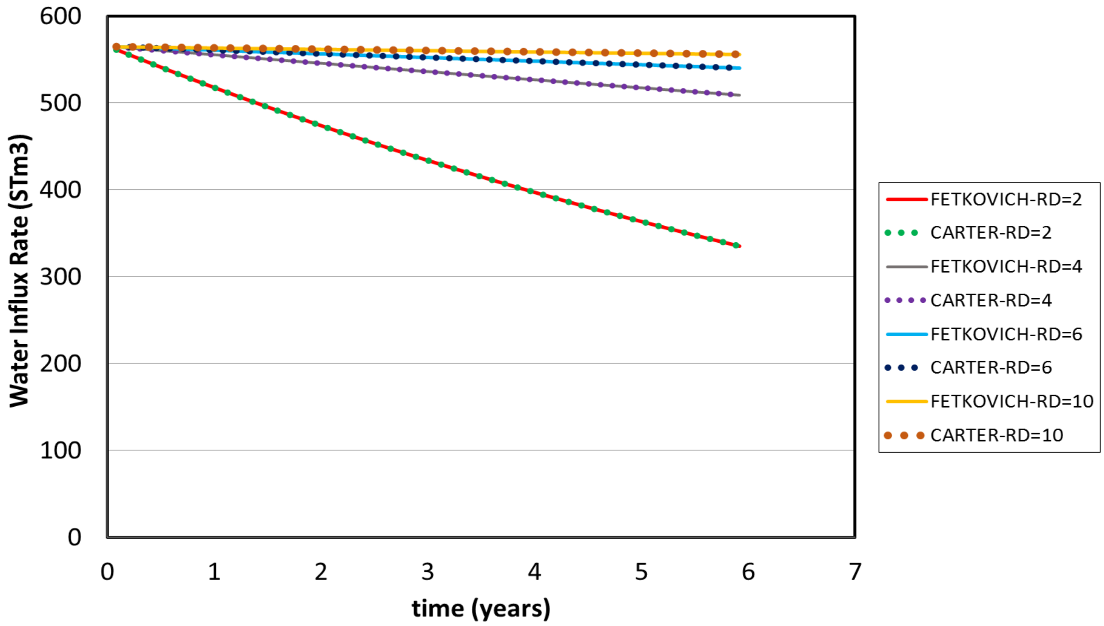

Fig. 5-18. Water Influx Rate for a net pay value of 10000m.

The discrepancies while changing the net pay values into the system as the pressure remained constant did not generate strong deviations into the performance of the aquifer models. Accordingly, both models tended to follow the same trend when they were applied, obtaining congruent results. However, as it could be observed, as the aquifer net pay increased the cumulative water influx did not have enough time to reach the stabilized part of graph, in which can be observed the maximum quantity of water depleted in the process. Nevertheless, the water influx rate tends to stabilize in time for large aquifer radius, which represents its maintenance till the pressure disturbance, generated by the production well, reaches the aquifer boundaries. Furthermore, the higher quantity of water entering into the reservoir as radius aquifer increased could also be observed.

5.1.2 Constant Flow Rate

For a constant flow rate of 100m^3/day, it will be showed the effects on changing parameters as permeability and net pay values.

5.1.2.1 Changing Permeability values

During the tests carried out to verify the behavior of the models while changing permeability values in a constant rate process was founded that the discrepancies between them were so small. Additionally, for all the range of aquifer radius, the trend of the aquifer models do not show any strong variation between them, thus it will be presented a specific test carried out for a precise value of aquifer radius and not for all of them, as it was showed previously.

For a reservoir with 500 meters of external radius and an aquifer radius of 750 meters (r_D=1,5); it has been tested the behavior of the aquifer models when they were exposed to changes in the permeability of the aquifer system. As for instance, a range of permeability values has been selected:

For permeability equal to 1 mD:

a)Cumulative Influx Rate

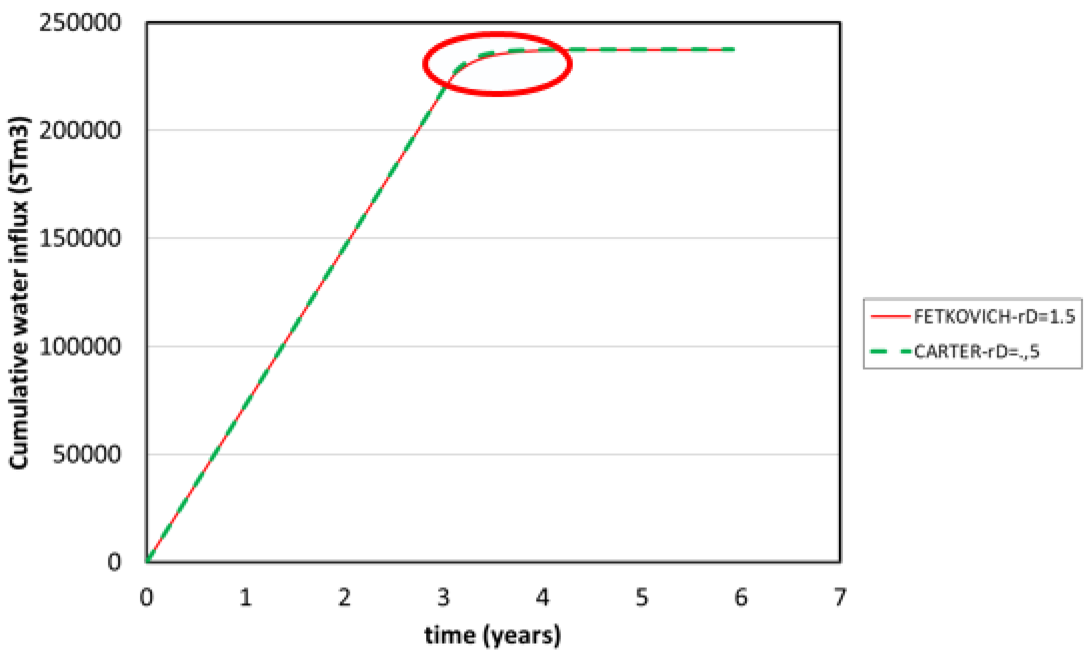

Fig. 5-19. Cumulative Water Influx for k=1mD at Constant Flow Rate.

b) Water Influx Rate

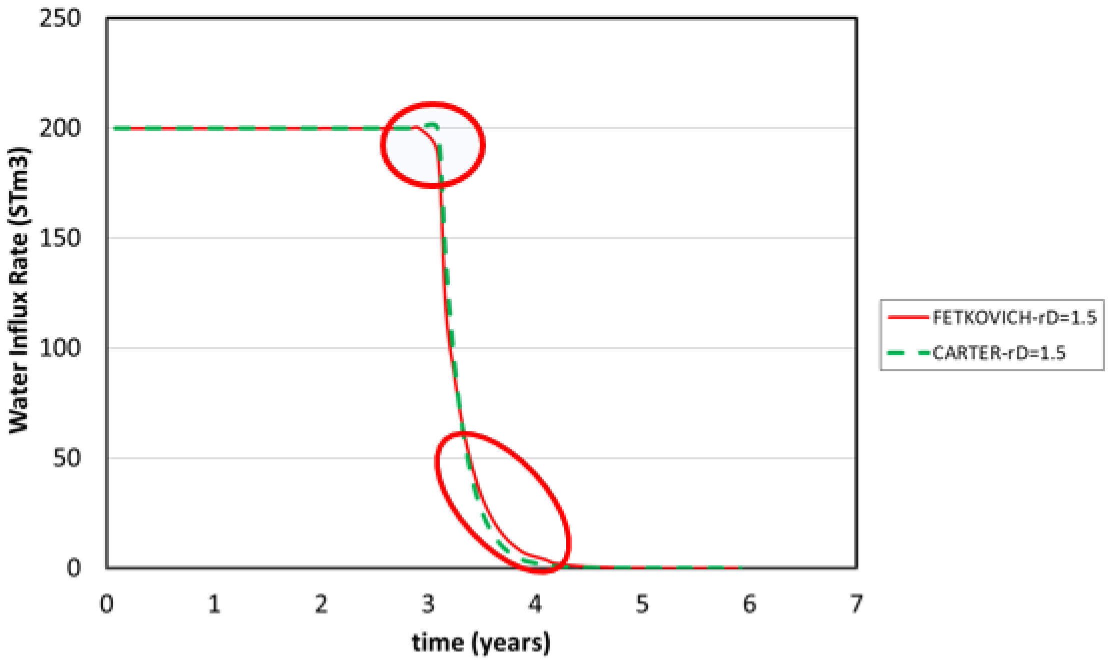

Fig. 5-20. Water Influx Rate for k=1mD at Constant Flow Rate.

For a permeability equal to 10 mD

a) Cumulative Water Influx

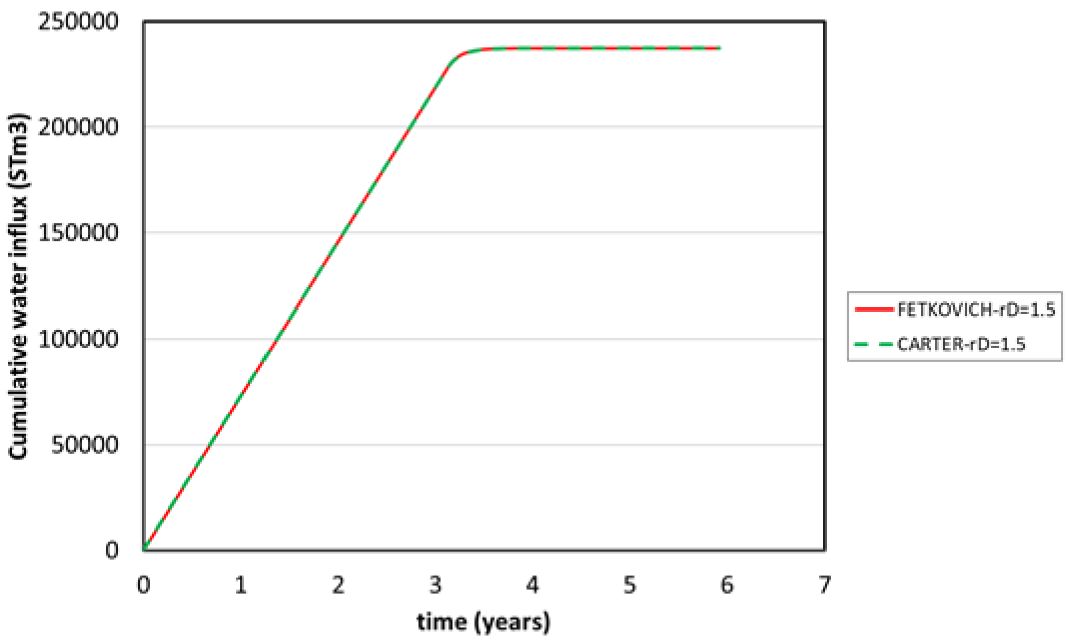

Fig. 5-21. Cumulative Water Influx for k=10mD at Constant Flow Rate.

b) Water influx Rate

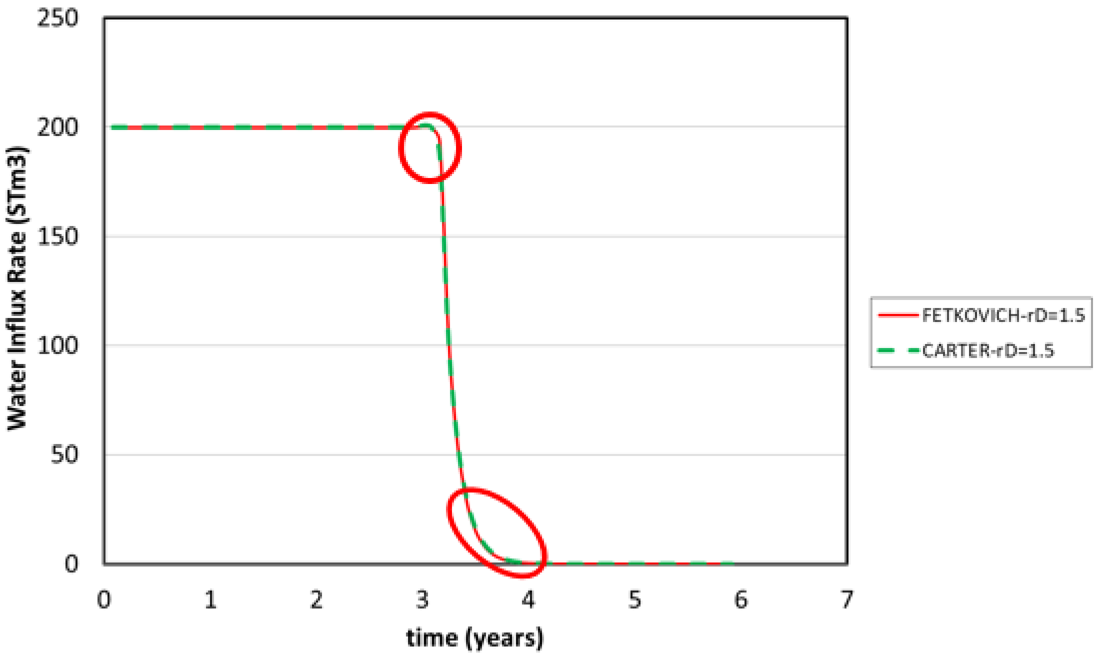

Fig. 5-22. Water Influx Rate for k=10mD at Constant Flow Rate.

For a permeability of 100mD

a) Cumulative Water Influx

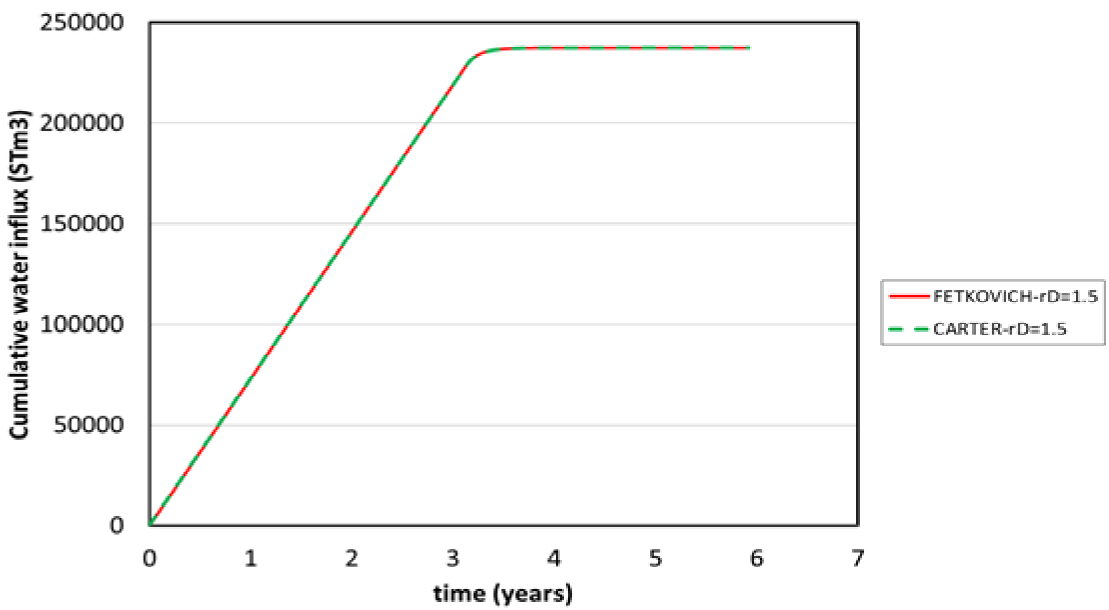

Fig. 5-23. Cumulative Water Influx for k=100mD at Constant Flow Rate.

b) Water Influx Rate

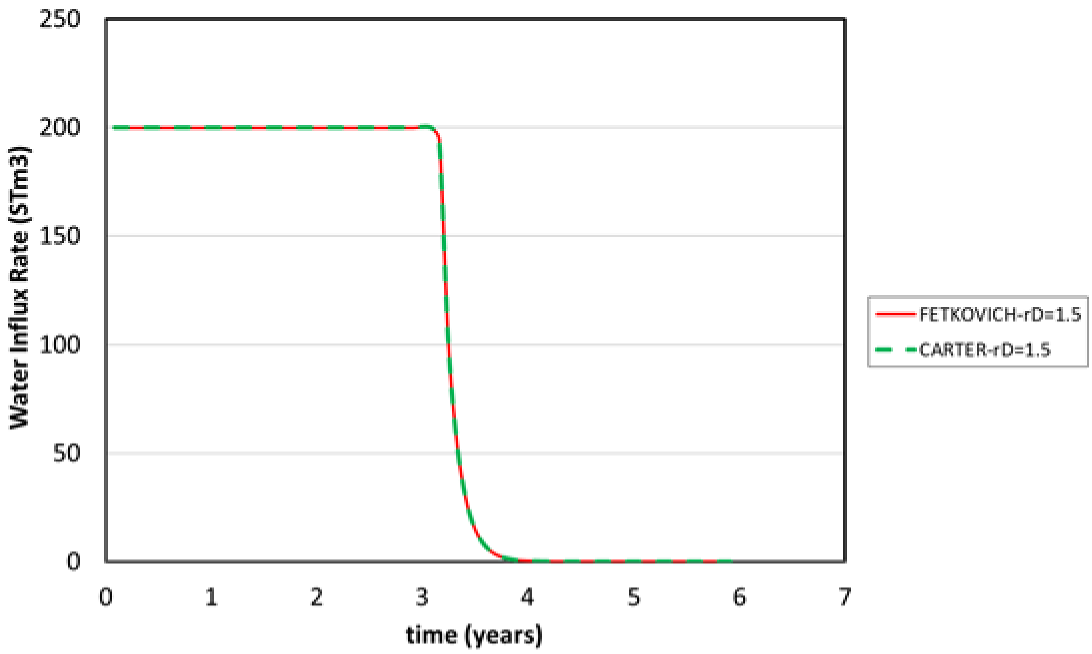

Fig. 5-24. Water Influx Rate for k=100mD at Constant Flow Rate.

Even whether in this test could be observed some discrepancies between the models when permeability values were decreased, it is important to mention that the fluctuations in the permeability range when the flow rate of the system remained constant, was not a strong factor for generating divergence in the performance of the aquifer models. However, it is important to clarify that differences between aquifer model responses could be appreciated for reservoirs with small values of permeability.

5.1.2.2 Changing Net Pay values

For a net pay value of 200 meters:

a) Cumulative Water Influx:

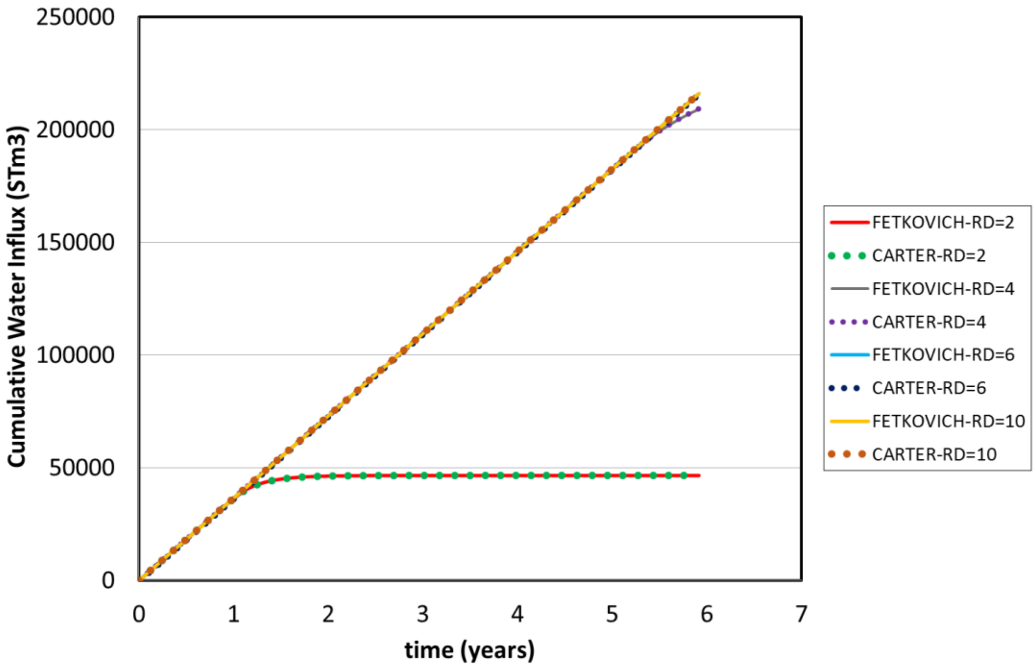

Fig. 5-25. Cumulative Water Influx for net pay value of 200m at Constant Flow Rate.

b) Water Influx Rate:

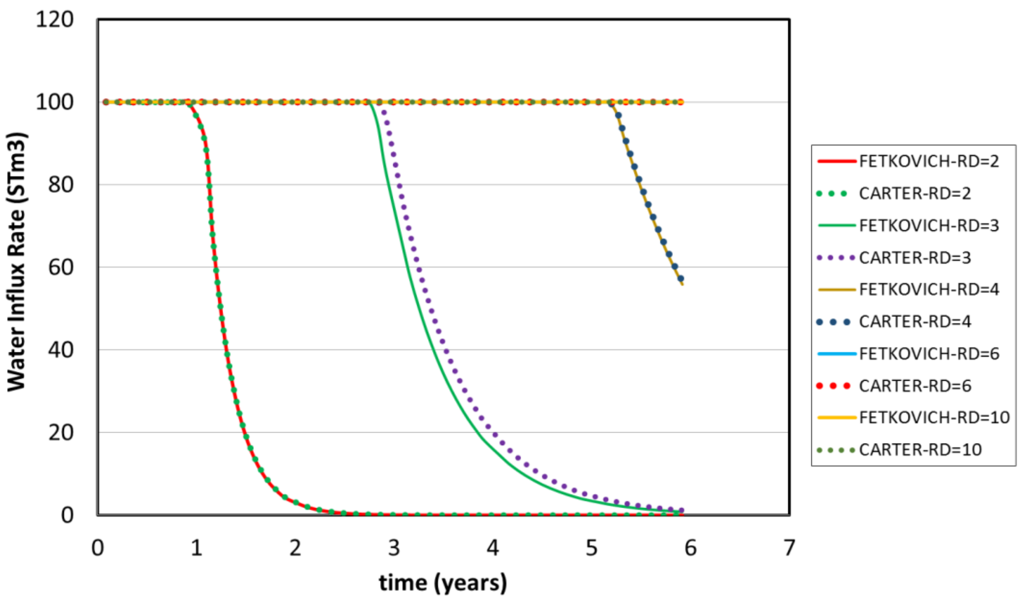

Fig. 5-26. Water Influx Rate for net pay value of 200m at Constant Flow Rate.

For a net pay value of 500 meters:

a) Cumulative Water Influx:

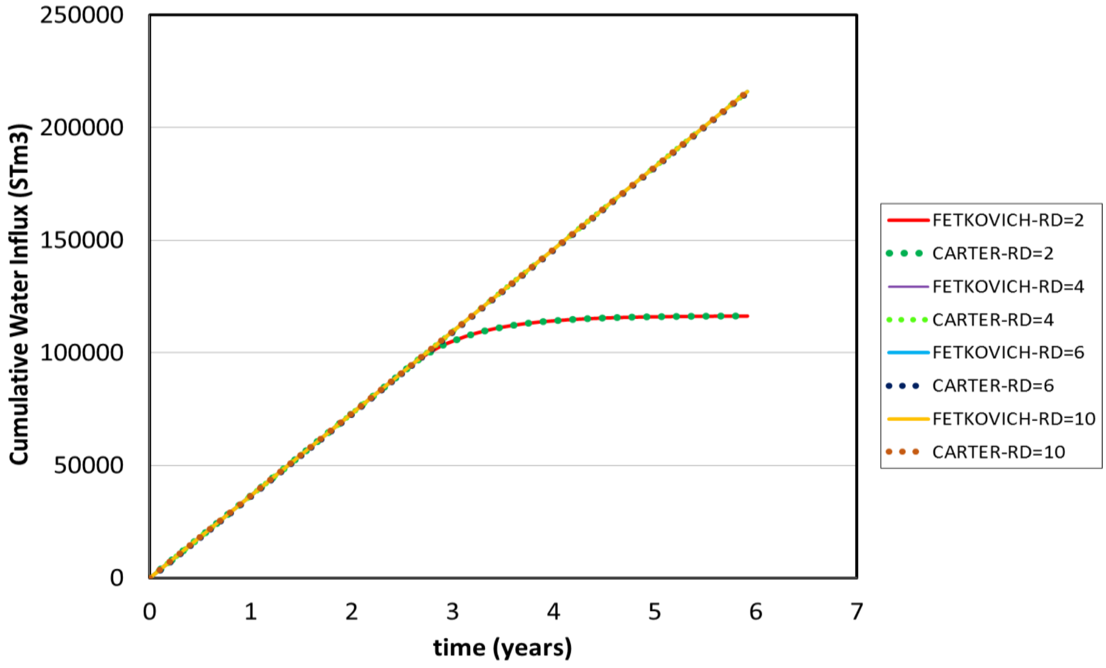

Fig. 5-27. Cumulative Water Influx Rate for net pay value of 500m at Constant Flow Rate.

b) Water Influx Rate:

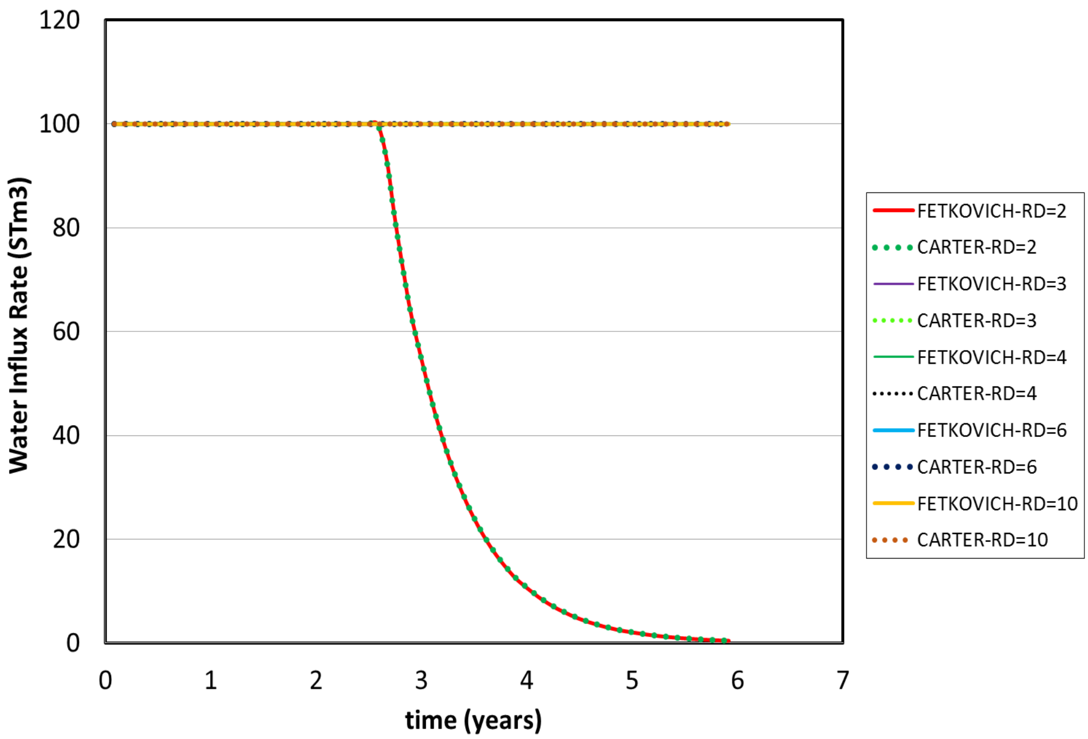

Fig. 5-28. Water Influx Rate for net pay value of 500m at Constant Flow Rate.

For a net pay value of 1000 meters:

a) Cumulative Water Influx:

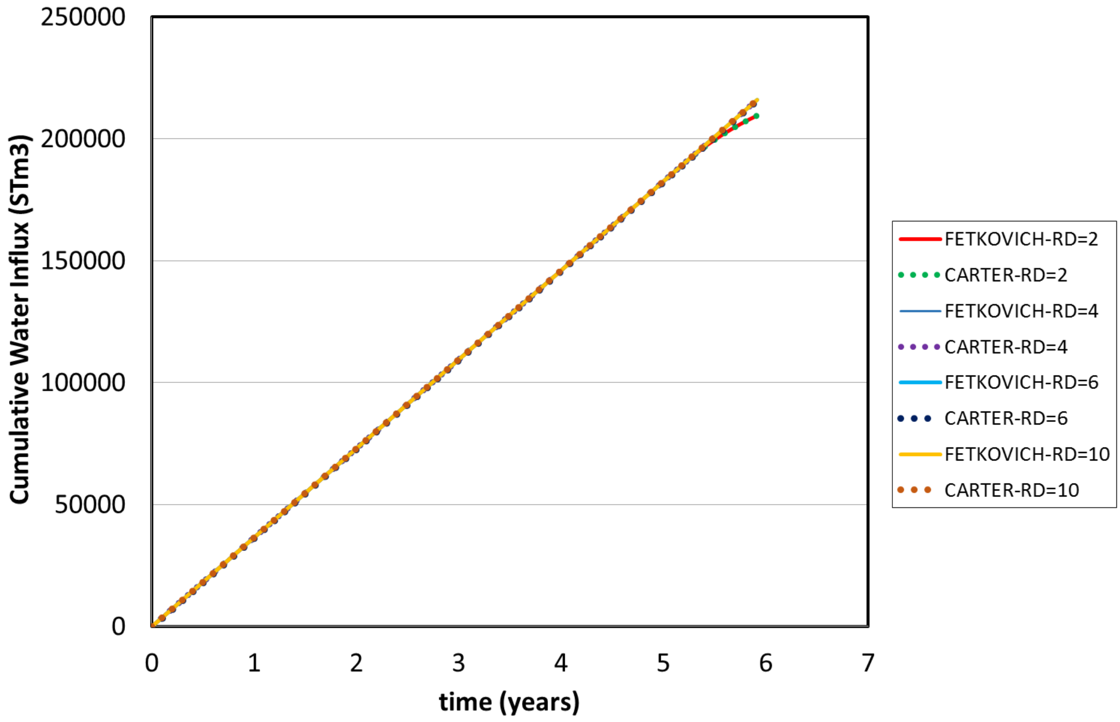

Fig. 5-29. Cumulative Water Influx for net pay value of 1000m at Constant Flow Rate.

b) Water Influx Rate:

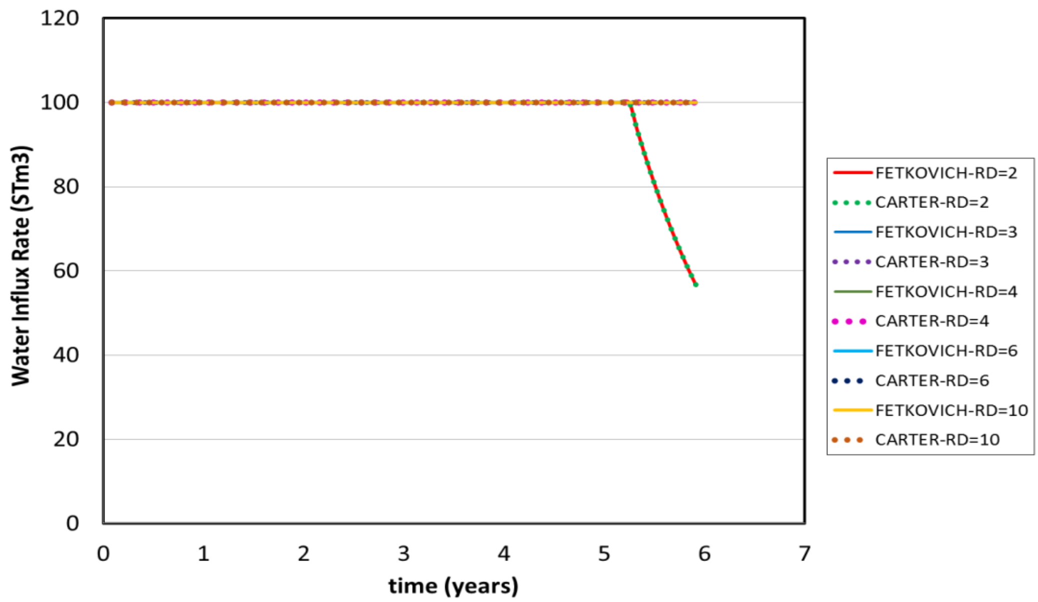

Fig. 5-30. Water Influx Rate for net pay value of 1000m at Constant Flow Rate.

For a net pay value of 10000 meters:

a) Cumulative Water Influx:

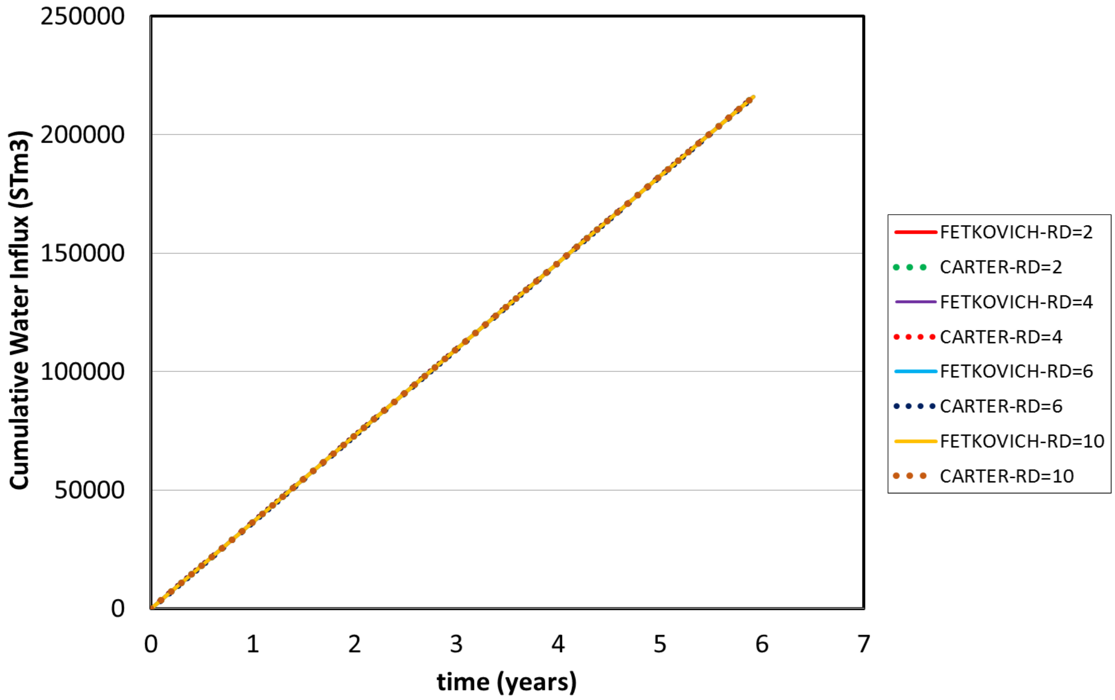

Fig. 5-31. Cumulative Water Influx for net pay value of 10000m at Constant Flow Rate

b) Water Influx Rate:

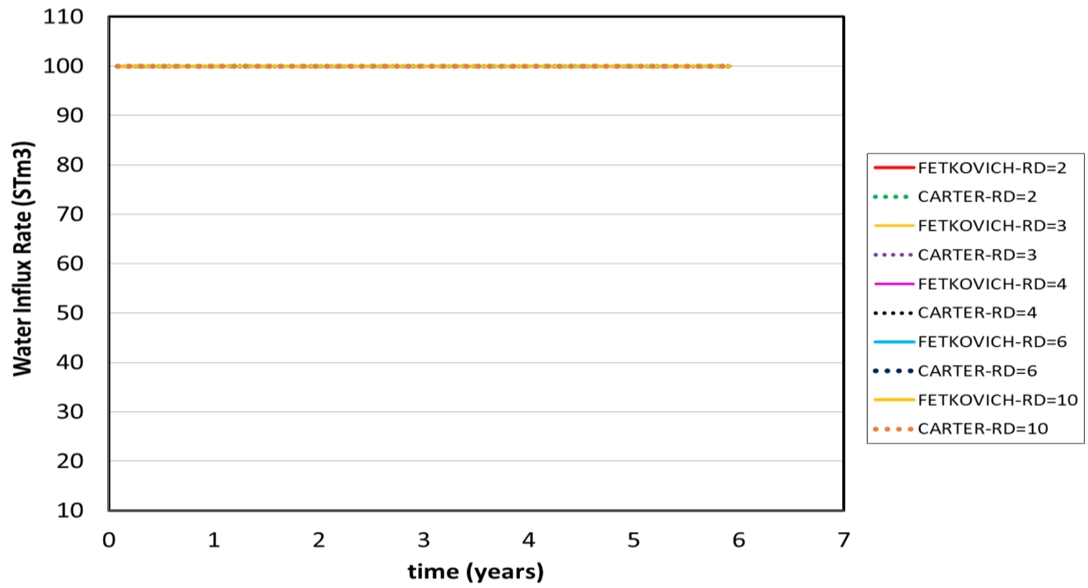

Fig. 5-32. Water Influx Rate for net pay value of 10000m at Constant Flow Rate.

5.1.3 Analytical vs Numerical Aquifer Model.

In order to compare the behavior of the analytical models against the numerical aquifer performance a special test was carried out, observing a similar scenario of the previous base case established in this study, against a numerical aquifer that maintain the same control and geological characteristics as follow:

- Type of aquifer: Radial
- Permeability = 100mD.
- Dimensions: 2 cell 50x50x100 meters.
- Pressure at the datum: 300 bars.
- Porosity: 20%.
- Compressibility: 3.98x10-5.
- Thickness: 200m.
- Reservoir Radius: 500m.
- Datum Depth: 3000m.

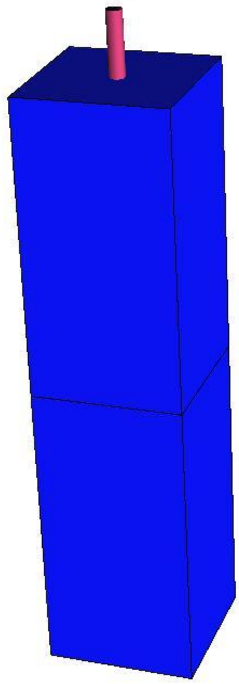

Fig. 5-33 Analytical vs Numerical aquifer model

The base case was modified from 1 cell to 2 cells for the grid section into the simulation tool, representing the new model for contrasting with the numerical model. For the numerical model was needed the calculation of the cross sectional area occupied by the aquifer as $AREA = \pi\,({r_a}^2 - {r_r}^2)$, where r_a represents the aquifer radius and r_r represents the reservoir radius. Accordingly, a different value of cross sectional area was calculated for each different aquifer radius previously proposed (chapter 4).

The graphs to compare Analytical versus Numerical aquifer model have been organized for an aquifer radius equal to 2000, 3000, 4000 and 5000 meters, equivalent to a drainage radius of rD=4, rD=6, rD=8 and rD=10, respectively.

For a drainage radius equal to rD=4 (aquifer radius of 2000 meters):

a) Cumulative Water Influx:

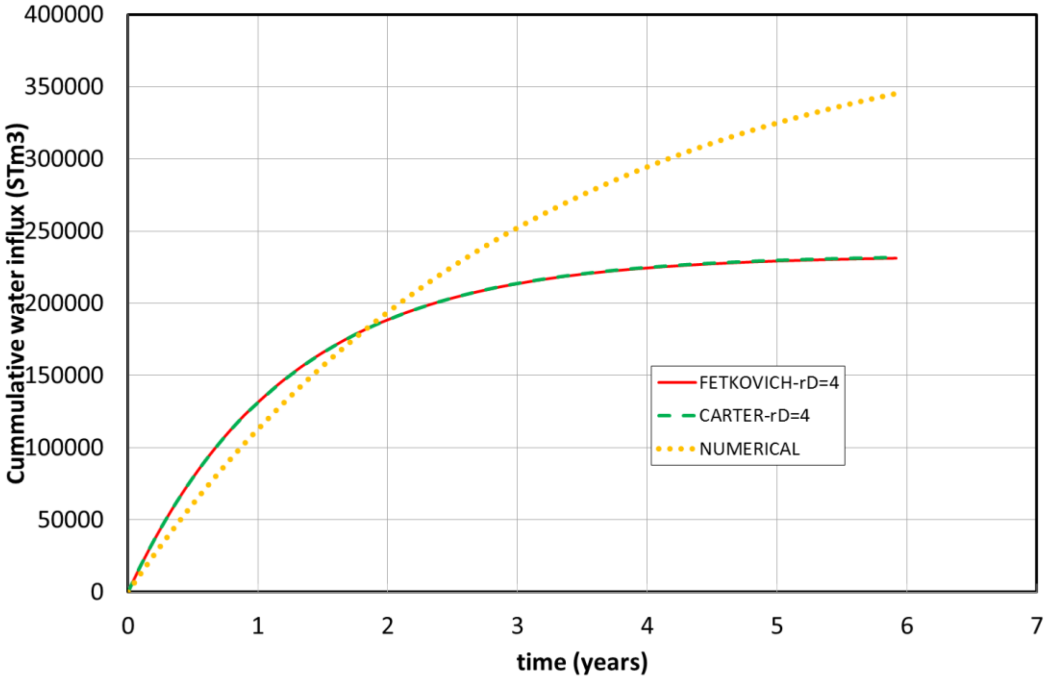

Fig. 5-34 Cumulative Water Influx for Numerical and Analytical aquifer models for rD=4.

b) Water Influx Rate:

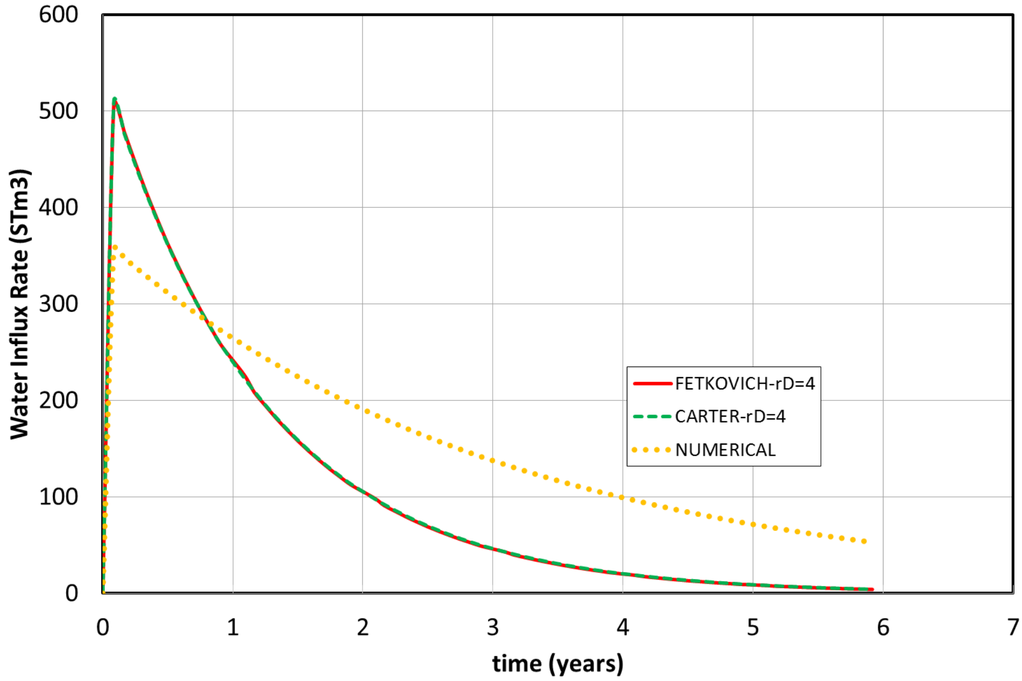

Fig. 5-35 . Water Influx Rate for Numerical and Analytical aquifer models for rD=4.

For a drainage radius equal to rD=6 (aquifer radius of 3000 meters):

a) Cumulative Water Influx:

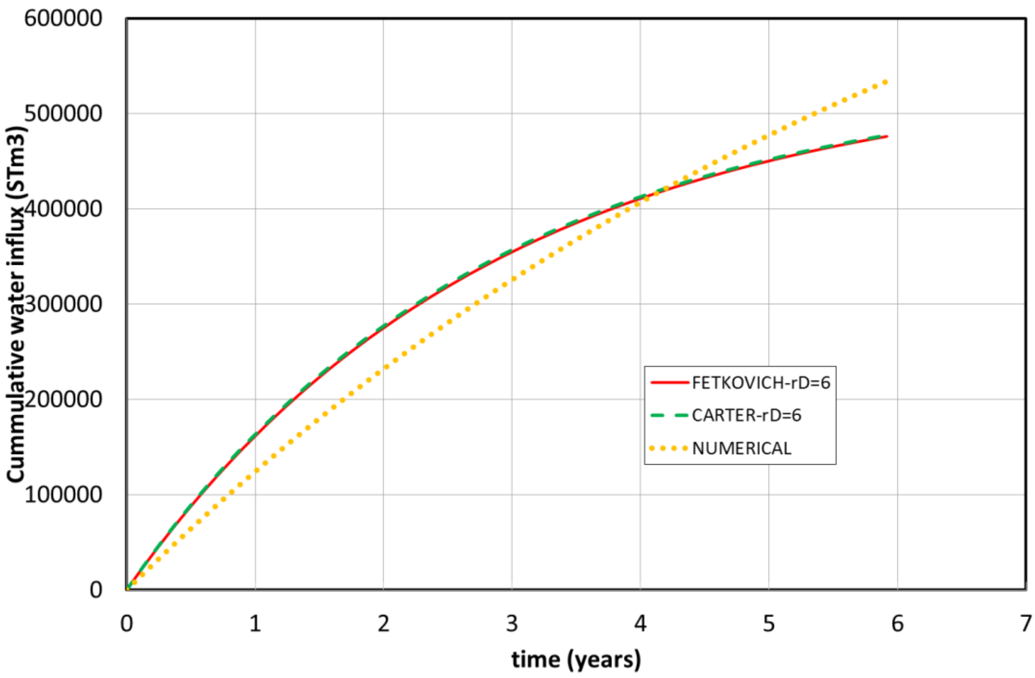

Fig. 5-36 Cumulative Water Influx for Numerical and Analytical aquifer models for rD=6.

b) Water Influx Rate:

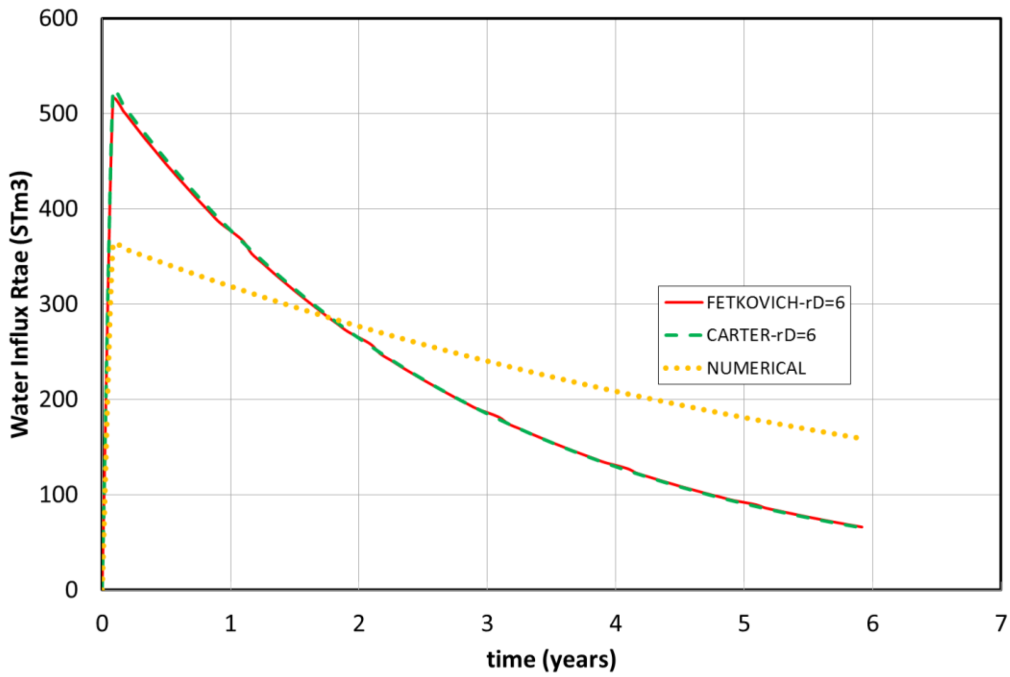

Fig. 5-37. Water Influx Rate for Numerical and Analytical aquifer models for rD=6.

For a drainage radius equal to rD=8 (aquifer radius of 4000 meters):

a) Cumulative Water Influx:

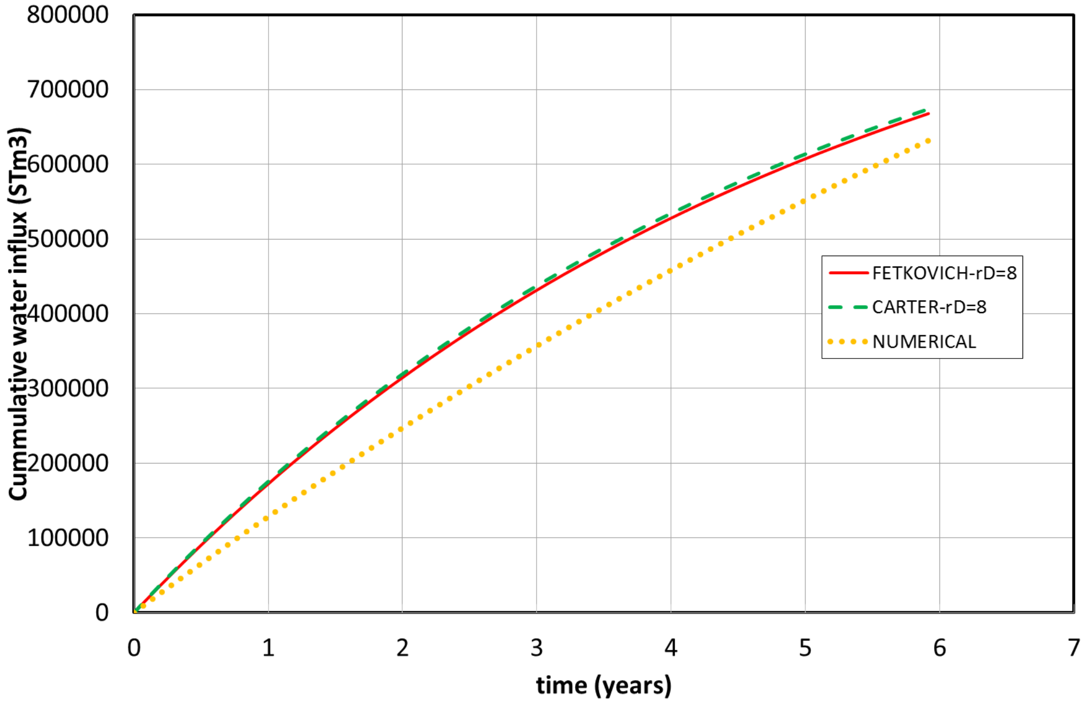

Fig. 5-38. Cumulative Water Influx for Numerical and Analytical aquifer models for rD=8

b) Water Influx Rate:

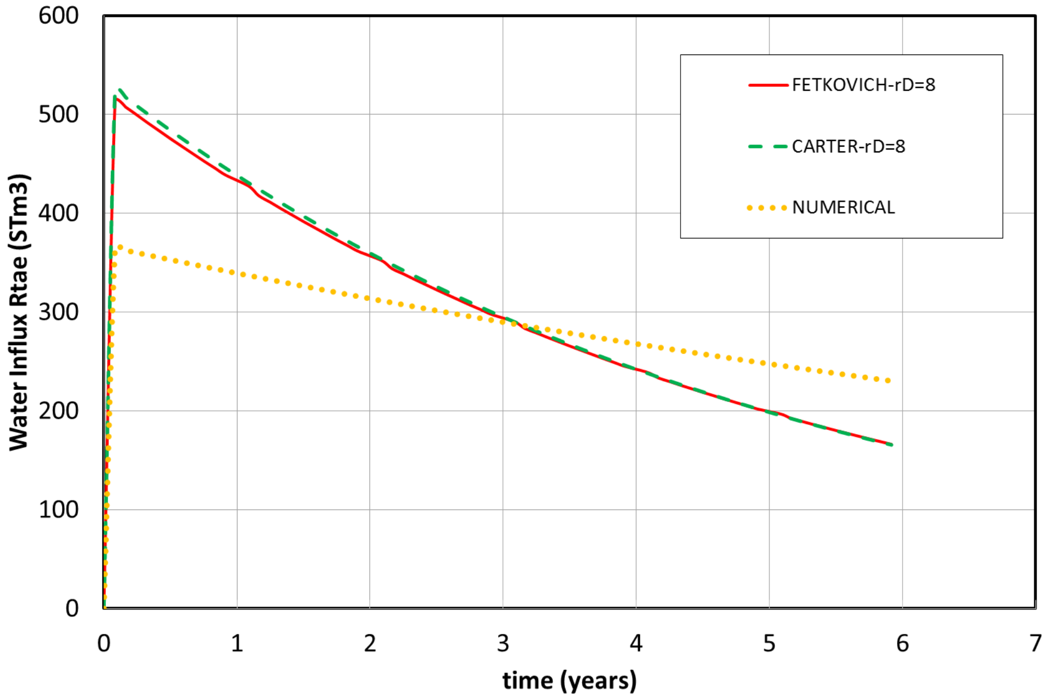

Fig. 5-39. Water Influx Rate for Numerical and Analytical aquifer models for rD=8.

For a drainage radius equal to rD=10 (aquifer radius of 5000 meters):

a) Cumulative Water Influx:

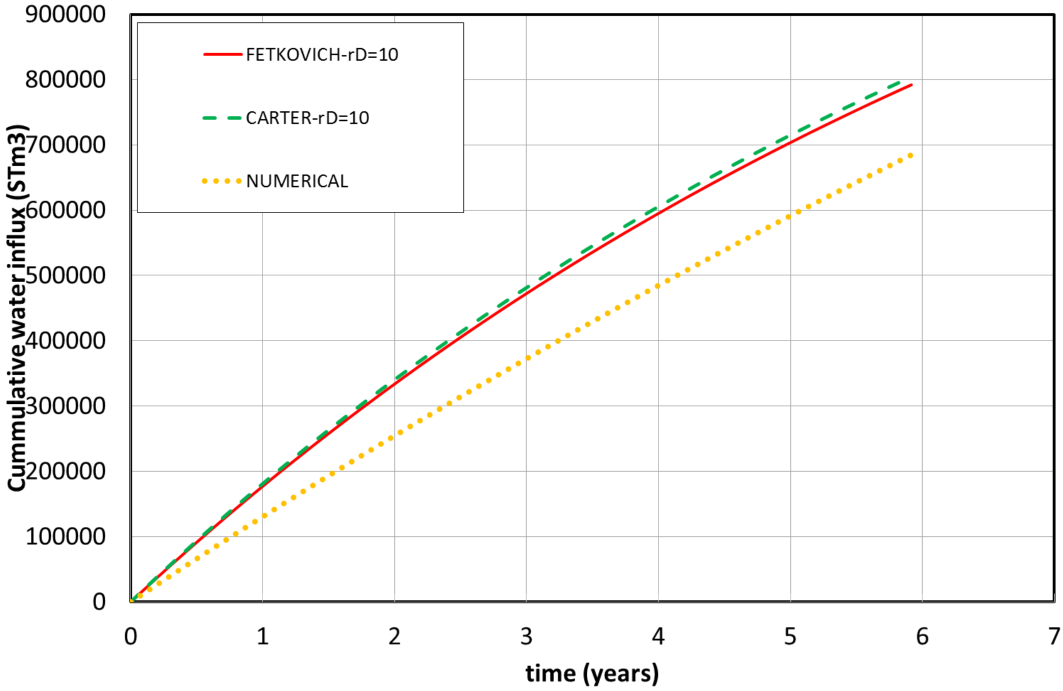

Fig. 5-40. Cumulative Water Influx for Numerical and Analytical aquifer models for rD=10.

b) Water Influx Rate:

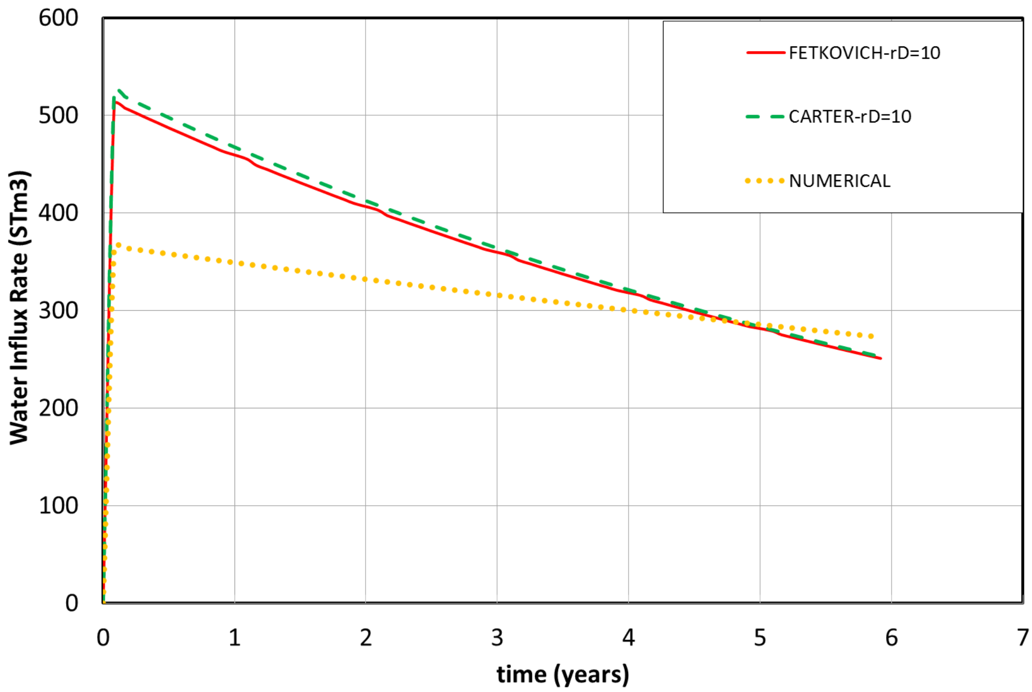

Fig. 5-41. Water Influx Rate for Numerical and Analytical aquifer models for rD=10.

As it could be appreciated in the previous graphs the aquifer and numerical models behavior drastically presented a dissimilar performance between them, even when they were simulating the same structure under the same conditions and production constraints (Bottom hole constant pressure).

Even when numerical and analytical aquifers were being run to represent the same structure under the same conditions, they did not apply the same equations to perform the aquifer behavior. Accordingly, the main difference between the models lies into the fact that numerical aquifer requires the calculation of the cross sectional area for a set of cells attached to the reservoir zone, and then imposing the boundary conditions and initial condition in the area selected for the model. In the other hand, the analytical solutions do not need the same amount of data and details as the numerical solutions, but they can still perform a good appreciation of the physical conditions in the area selected to represent the aquifer zone.

It is important to do mention to the fact that working with analytical models means that their solutions do not contain all details of an aquifer, but the main character of the aquifer may be produced. Moreover, analytical models have some particular characteristics, as for example, using Darcy's Law with conservation of mass to generate a partial differential equation while discretization of the domain selected is not carried out, as numerical aquifer does.

From the analysis between the behavior observed from the analytical and numerical models can be concluded that the models represented the aquifer performance in different ways due to principally their theoretical bases, and thus their different equations implemented to characterize an aquifer body.

Finally, having considered in the previous paragraphs the main characteristics of each aquifer model to better represent the performance of an aquifer body, could be suggested using numerical aquifer models for those cases in which is well defined the flow units from the aquifer into the reservoir, as for example, having wells well dispersed around the reservoir, and in the aquifer, as well as. In the other hand, could be recommended analytical aquifer models in those cases where do not exist well information in the aquifer, as for instance when a set of wells are clustered away from the aquifer zone.

6. CONCLUSION AND RECOMMENDATIONS

This investigation was conducted to better understand the behavior and performances of aquifer models using the simulator Eclipse 100 (a Schlumberger simulator tool), considering the necessity that the oil and gas industry have to appreciate and characterize aquifer bodies, when those surround a hydrocarbon reservoir. In order to focus exclusively on the behavior of the considered models, an extremely reduced aquifer structure was structured and analyzed.

The Carter-Tracy and Fetkovich aquifer models were initially subjected to an adequate calibration of the models, following with the creation of a base model to be used as a reference. From the tests carried out could be observed the main differences found between them, which were strictly related to the flow regime that governs the aquifer model functionality, and thus finding drastically changes into the performances of the models when alterations in permeability values were carried out. Furthermore, changes implemented in the net pay values were not a major factor to create discrepancies between the models.

Moreover, the contrast between analytical and numerical aquifer models was demonstrated, observing important discrepancies while they were representing the same aquifer system, since equivalent results were not found. The main reasons at which are attributed these differences lies in the implementation of different equations by each aquifer model.

This research highlights the existing differences in the considered aquifer models. It should be pointed out that in real situation it is not possible to establish a priori which model is better to use to represent as closely as possible the analyzed aquifer. A possible way is to perform a sensitivity analysis in order to detect which of the considered models match closely to the historical production data.

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